Bottom-Up Parsing II

CS143

Lecture 8

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Slide design by Prof. Alex Aiken, with modifications

Review: Shift-Reduce Parsing

Bottom-up parsing uses two actions:

Shift

ABC | xyz $\Rightarrow$ ABCx | yz

Reduce

Cbxy | ijk $\Rightarrow$ CbA | ijk

Recall: The Stack

- Left string can be implemented by a stack
  - Top of the stack is the |
- Shift pushes a terminal on the stack
- Reduce
  - pops 0 or more symbols off of the stack
    - production rhs
  - pushes a non-terminal on the stack
    - production lhs

Key Issue

- How do we decide when to shift or reduce?

- Example grammar:

  E → T + E | T
  T → int * T | int | (E)

- Consider step int | * int + int
  - We could reduce by T → int giving T | * int + int
  - A fatal mistake!
    - No way to reduce to the start symbol E

Definition: Handles

- Intuition: Want to reduce only if the result can still be reduced to the start symbol

- Assume a rightmost derivation

$$S \rightarrow^* \alpha X \omega \rightarrow \alpha\beta\omega$$

- Then $X \rightarrow \beta$ in the position after α is a handle of $\alpha\beta\omega$

- Can and must reduce at handles

Handles (Cont.)

- Handles formalize the intuition
  - A handle is a string that can be reduced and also allows further reductions back to the start symbol (using a particular production at a specific spot)

- We only want to reduce at handles

- Note: We have said what a handle is, not how to find handles

Important Fact #2

Important Fact #2 about bottom-up parsing:

In shift-reduce parsing, handles appear only at the top of the stack, never inside

Why?

- Informal induction on # of reduce moves:
- True initially, stack is empty
- Immediately after reducing a handle
  - right-most non-terminal on top of the stack
  - next handle must be to right of right-most non-terminal, because this is a right-most derivation
  - Sequence of shift moves reaches next handle

Summary of Handles

- In shift-reduce parsing, handles always appear at the top of the stack

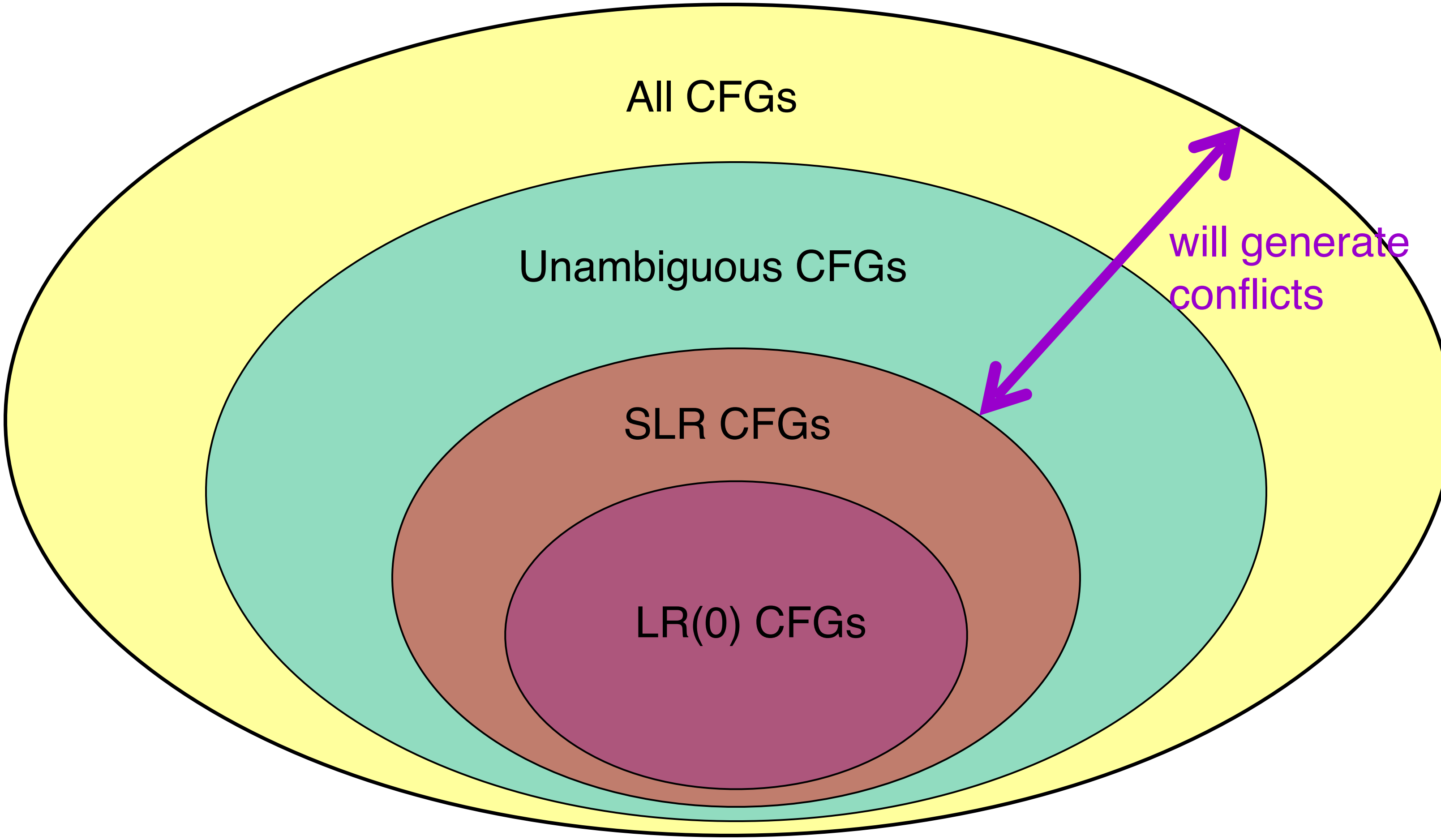
- Handles are never to the left of the rightmost non-terminal
  - Therefore, shift-reduce moves are sufficient; the I need never move left

- Bottom-up parsing algorithms are based on recognizing handles

Recognizing Handles

- There are no known efficient algorithms to recognize handles
- Solution: use heuristics to guess which stacks are handles
- On some CFGs, the heuristics always guess correctly
  - For the heuristics we use here, these are the SLR grammars
  - Other heuristics work for other grammars

Grammars


All CFGs
Unambiguous CFGs
SLR CFGs
LR(0) CFGs
will generate
conflicts

Viable Prefixes

- It is not obvious how to detect handles

- At each step the parser sees only the stack, not the entire input; start with that . . .

α is a viable prefix if there is an ω such that $\alpha|\omega$ is a state of a shift-reduce parser

Huh?

- What does this mean? A few things:
  - A viable prefix does not extend past the right end of the handle
  - It’s a viable prefix because it is a prefix of the handle
  - As long as a parser has viable prefixes on the stack no parsing error has been detected

Important Fact #3

Important Fact #3 about bottom-up parsing:

For any SLR grammar, the set of viable prefixes is a regular language

Important Fact #3 (Cont.)

- Important Fact #3 is non-obvious
- We show how to compute automata that accept viable prefixes

Items

- An item is a production with a "." somewhere on the rhs, denoting a focus point

- The items for $T \rightarrow (E)$ are

  $T \rightarrow .(E)$
  $T \rightarrow (.E)$
  $T \rightarrow (E.)$
  $T \rightarrow (E).$

Items (Cont.)

- The only item for $X \rightarrow \varepsilon$ is $X \rightarrow .$
- Items are often called “LR(0) items”

Intuition

- The problem in recognizing viable prefixes is that the stack has only bits and pieces of the rhs of productions
  - If it had a complete rhs, we could reduce

- These bits and pieces are always prefixes of rhs of productions

$E \rightarrow T + E \mid T$
$T \rightarrow int * T \mid int \mid (E)$

Example

Consider the input (int)

- Then (E |) is a state of a shift-reduce parse
- (E is a prefix of the rhs of T → (E)
  - Will be reduced after the next shift
- Item T → (E.) says that so far we have seen (E of this production and hope to see)

Generalization

- The stack may have many prefixes of rhs's

  $\text{Prefix}_1\ \text{Prefix}_2 \ldots \text{Prefix}_{n-1}\ \text{Prefix}_n$

- Let Prefix_i be a prefix of rhs of $X_i \rightarrow \alpha_i$
  - Prefix_i will eventually reduce to X_i
  - The missing part of Prefix_{i-1} of α_{i-1} starts with X_i
  - i.e. there is a $X_{i-1} \rightarrow \text{Prefix}_{i-1}\ X_i\ \beta$ for some β

- Recursively, $\text{Prefix}_{k+1} \ldots \text{Prefix}_n$ eventually reduces to the missing part of α_k

$E \rightarrow T + E \mid T$
$T \rightarrow int * T \mid int \mid (E)$

An Example

Consider the string (int * int):

(int * | int) is a state of a shift-reduce parse

From top of the stack:

"ε" is a prefix of the rhs of $E \rightarrow T$

"(" is a prefix of the rhs of $T \rightarrow (E)$

"ε" is a prefix of the rhs of $E \rightarrow T$

"int *" is a prefix of the rhs of $T \rightarrow int * T$

An Example (Cont.)

$E \rightarrow T + E \mid T$
$T \rightarrow int * T \mid int \mid (E)$

The stack of items

$T \rightarrow int * .T$

$E \rightarrow .T$

$T \rightarrow (.E)$

Says

We've seen int * of $T \rightarrow int * T$

We've seen ε of $E \rightarrow T$

We've seen (of $T \rightarrow (E)$

Recognizing Viable Prefixes

Idea: To recognize viable prefixes, we must

- Recognize a sequence of partial rhs's of productions, where
- Each sequence can eventually reduce to part of the missing suffix of its predecessor

An NFA Recognizing Viable Prefixes

1. Add a new start production $S' \rightarrow S$ to G

2. The NFA states are the items of G
   – (Including the new start production)

3. For item $E \rightarrow \alpha.X\beta$ add transition

$$E \rightarrow \alpha.X\beta \ \rightarrow^{X} \ E \rightarrow \alpha X.\beta$$

4. For item $E \rightarrow \alpha.X\beta$ and production $X \rightarrow \gamma$ add

$$E \rightarrow \alpha.X\beta \ \rightarrow^{\varepsilon} \ X \rightarrow .\gamma$$

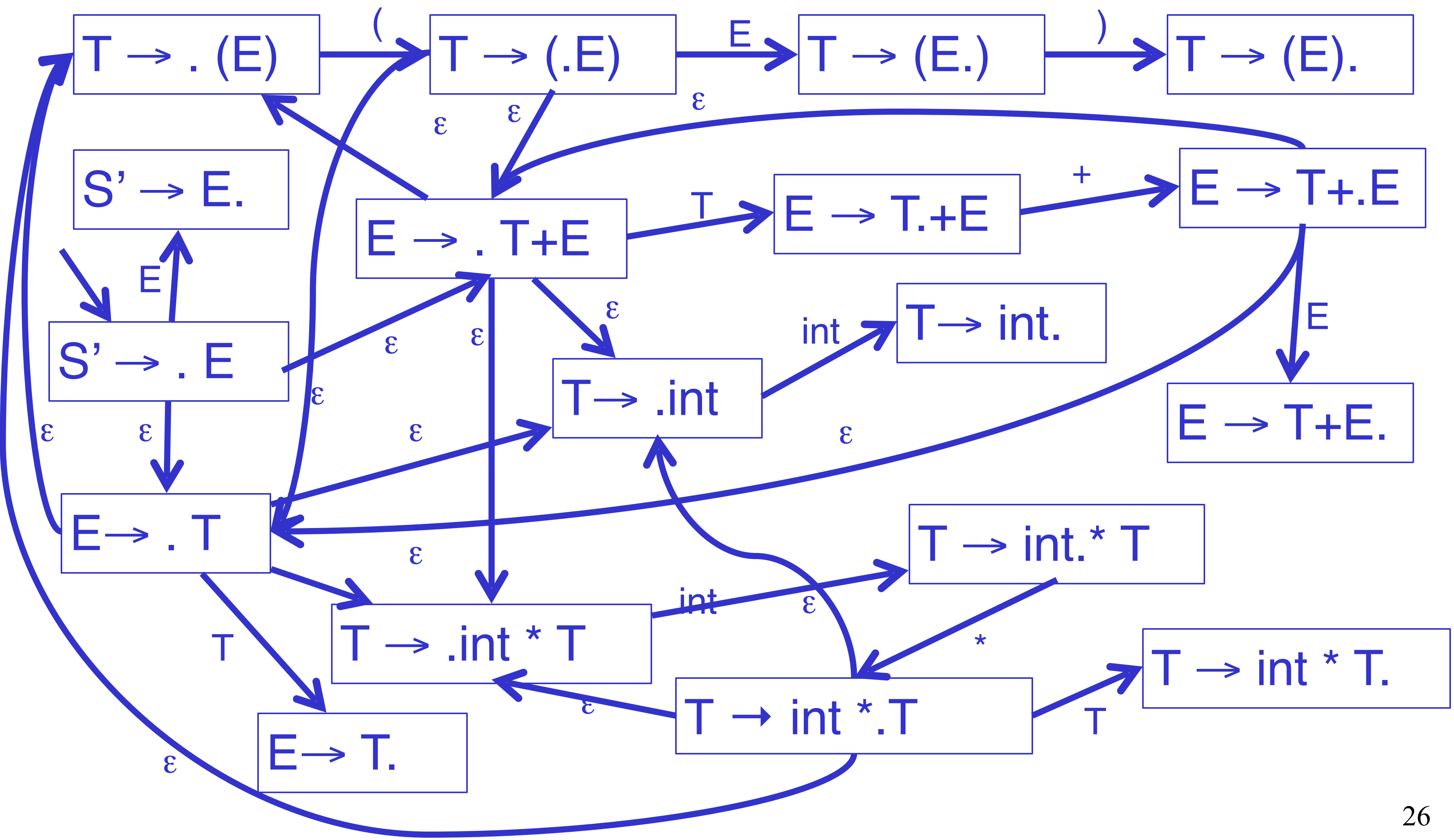
An NFA Recognizing Viable Prefixes (Cont.)

5. Every state is an accepting state

6. Start state is $S' \rightarrow .S$

E → T + E | T
T → int * T | int | (E)

NFA for Viable Prefixes



E → T + E | T
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NFA for Viable Prefixes

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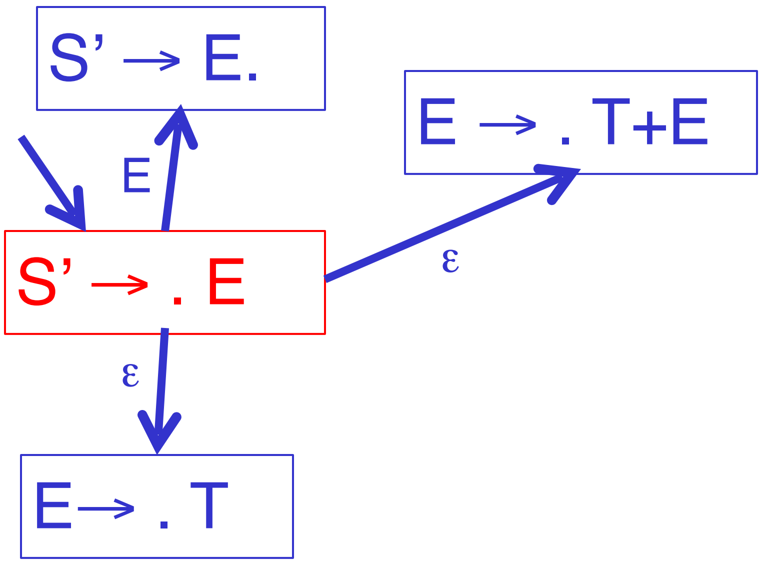

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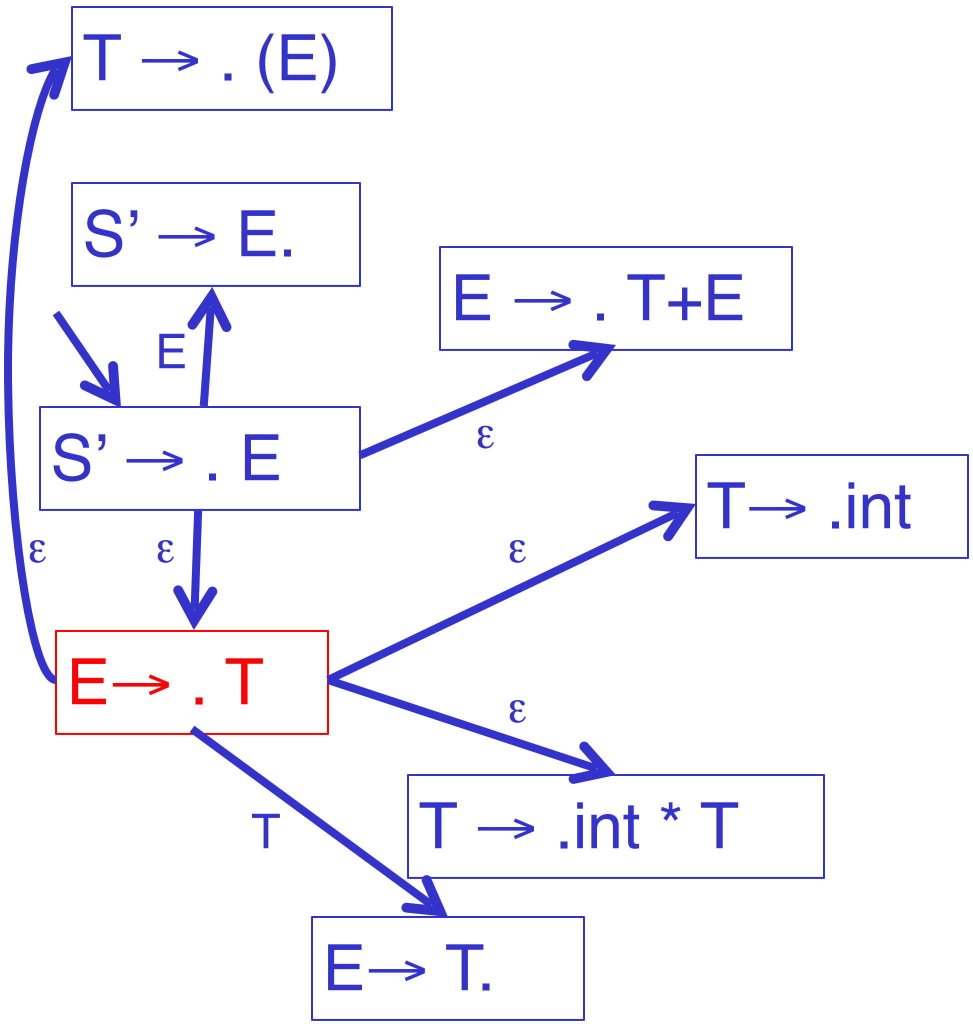
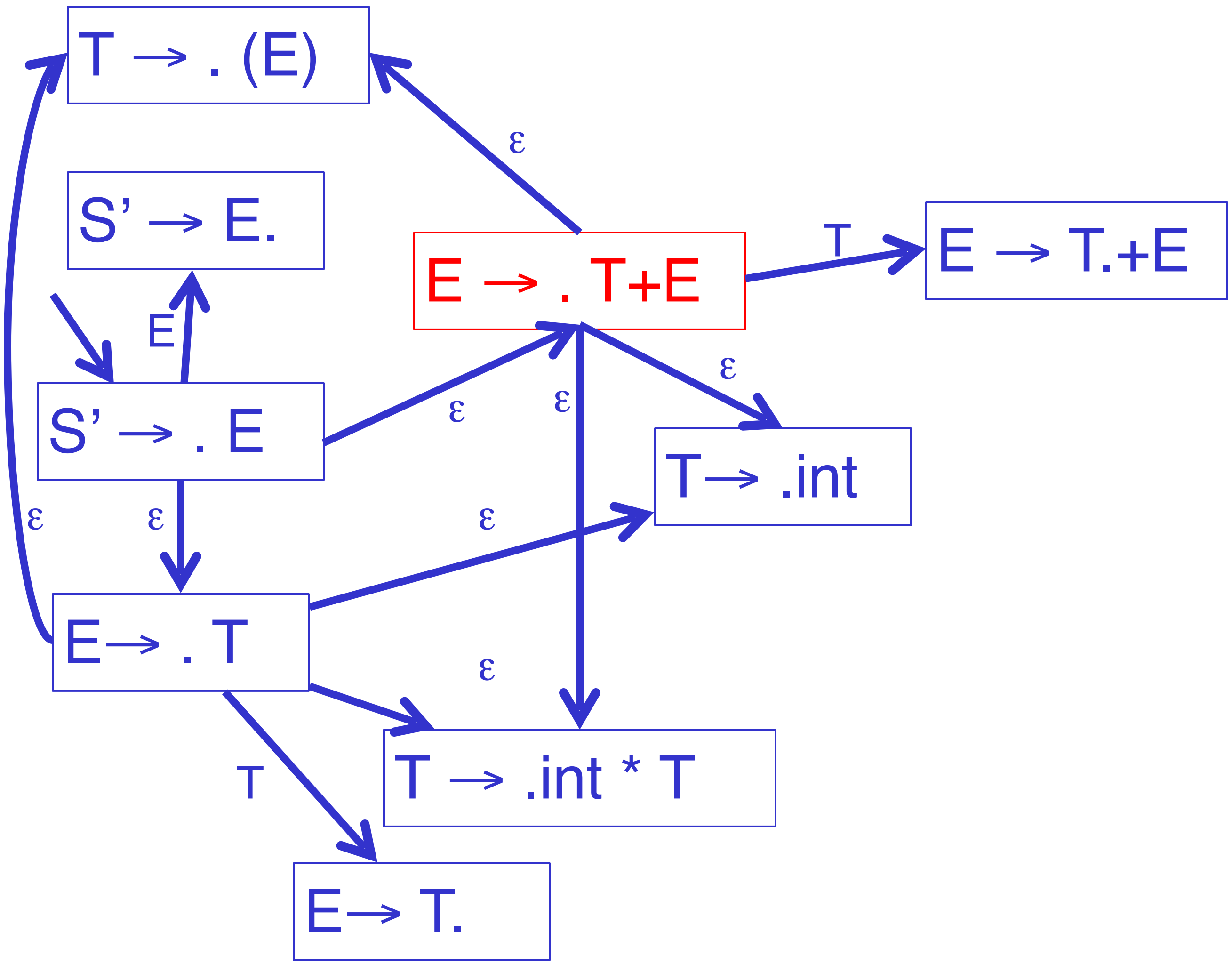


$E \rightarrow T + E \mid T$
$T \rightarrow int * T \mid int \mid (E)$

NFA for Viable Prefixes



E → T + E | T
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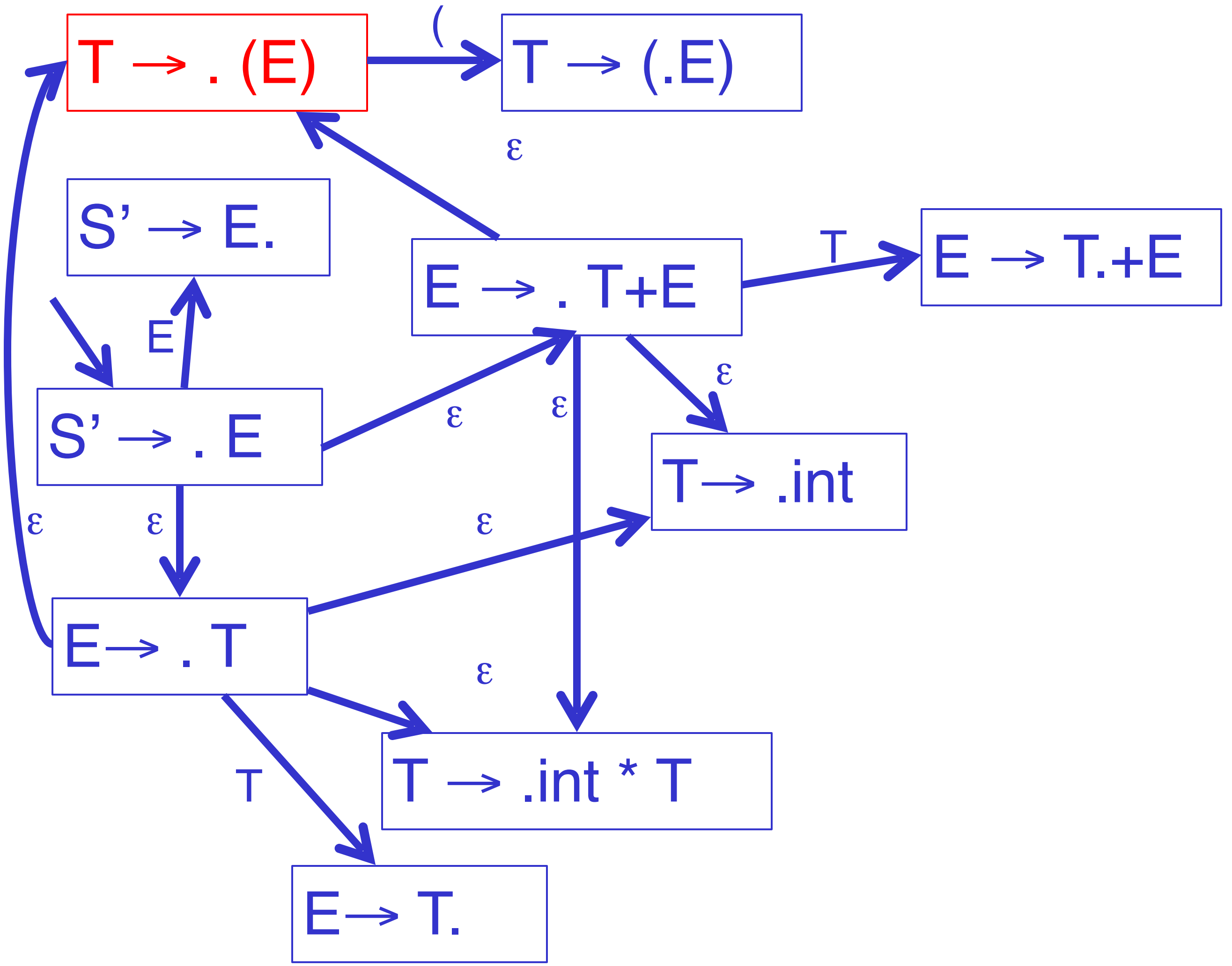
NFA for Viable Prefixes



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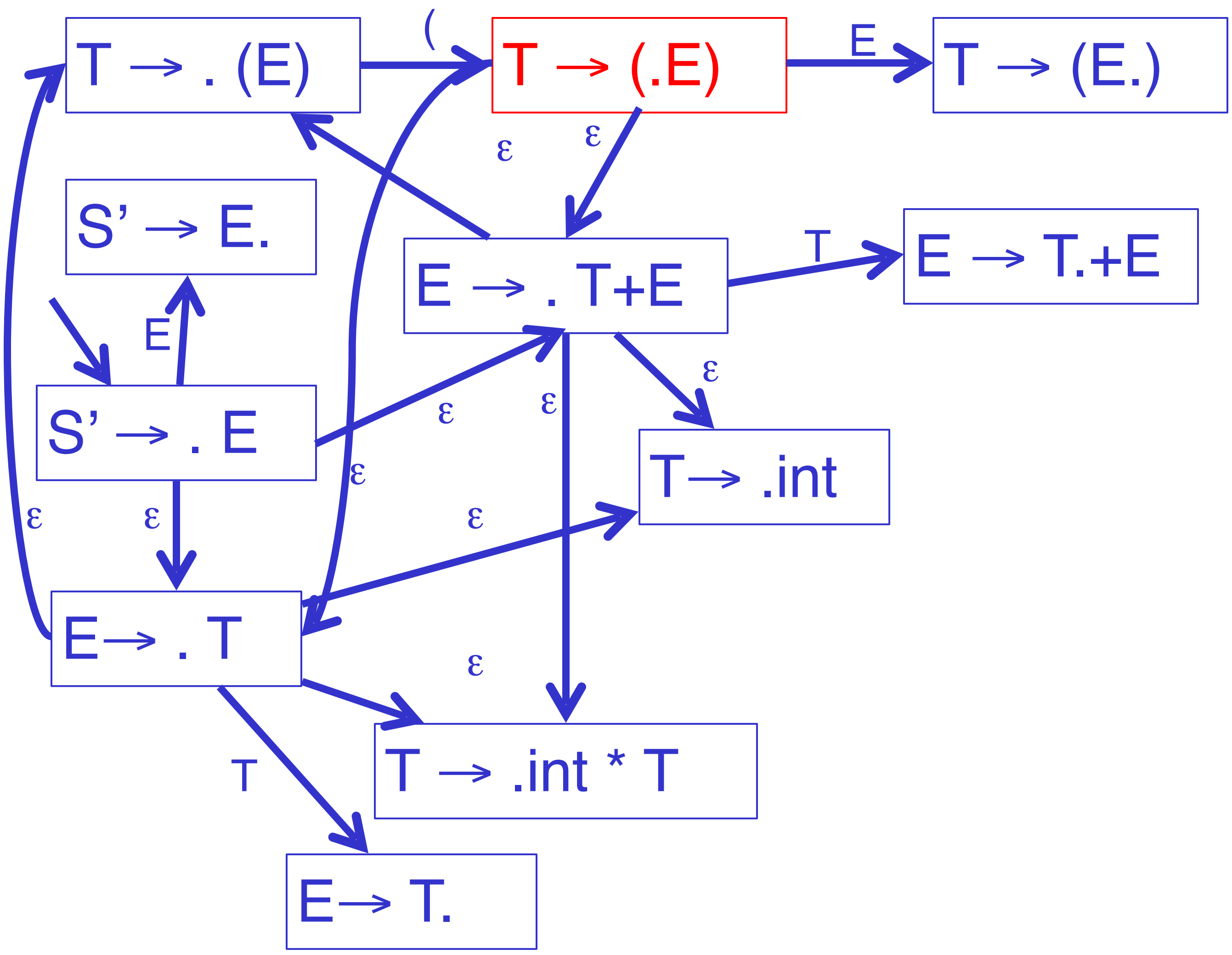
NFA for Viable Prefixes



E → T + E | T
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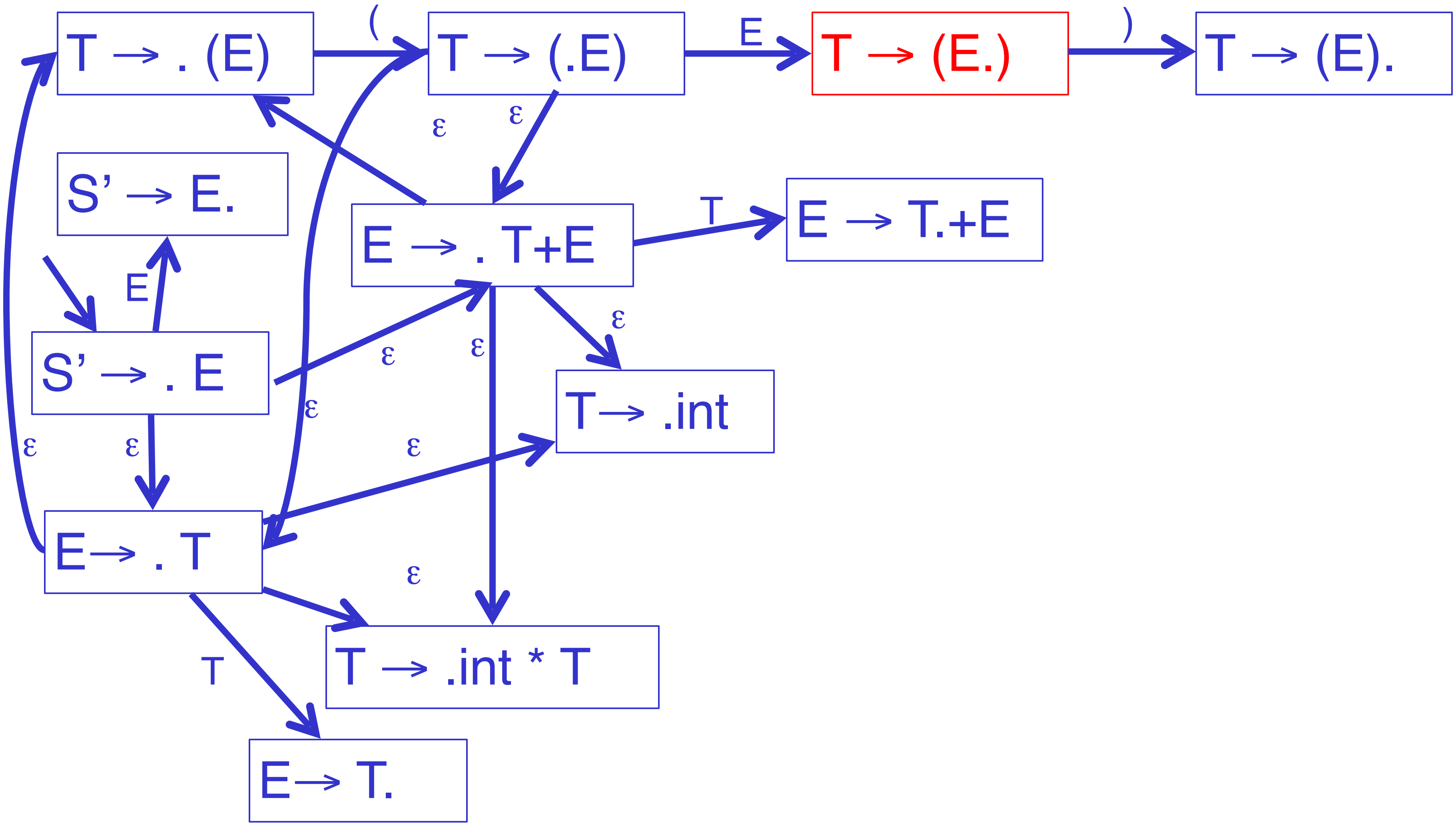

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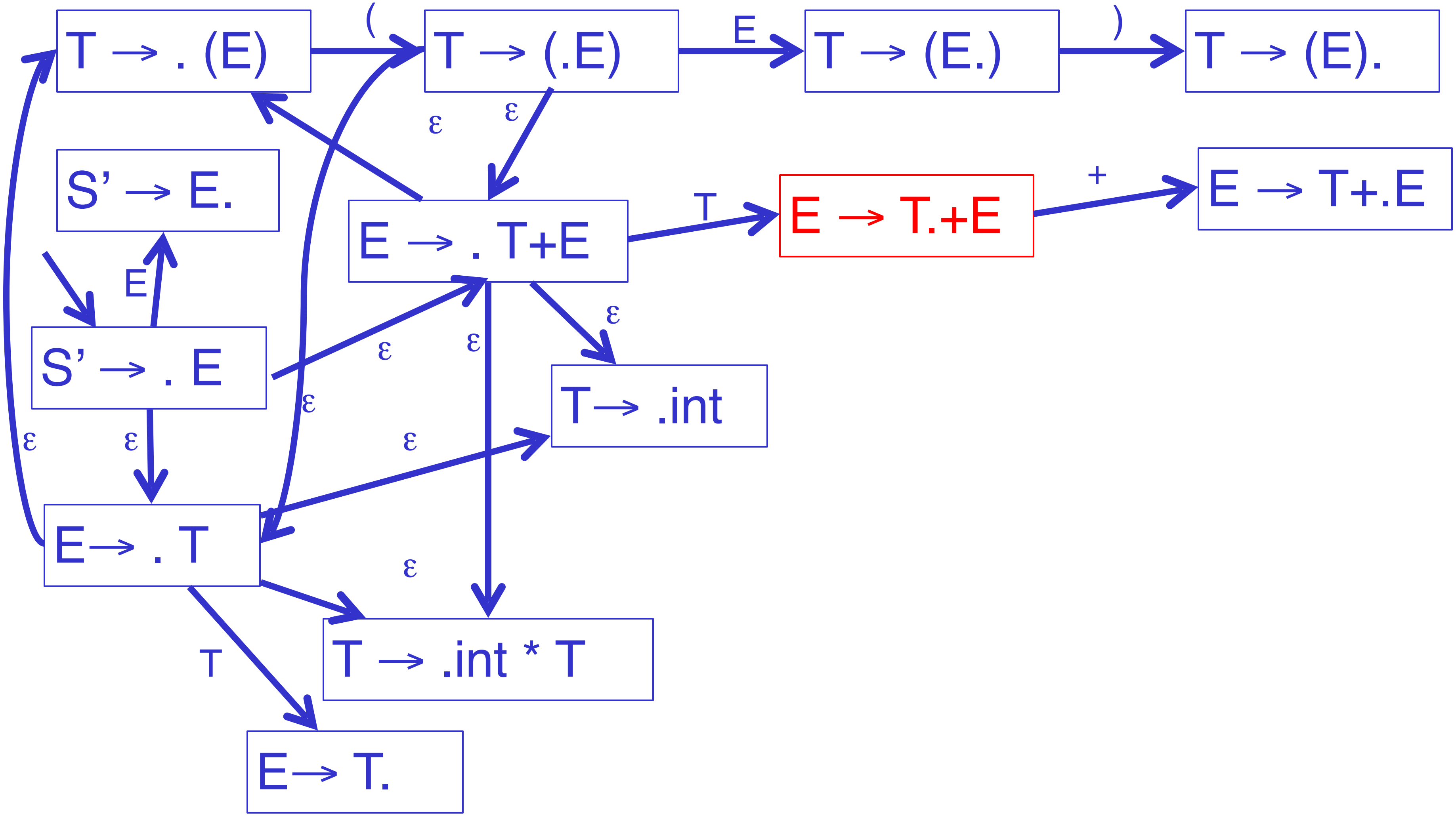
NFA for Viable Prefixes



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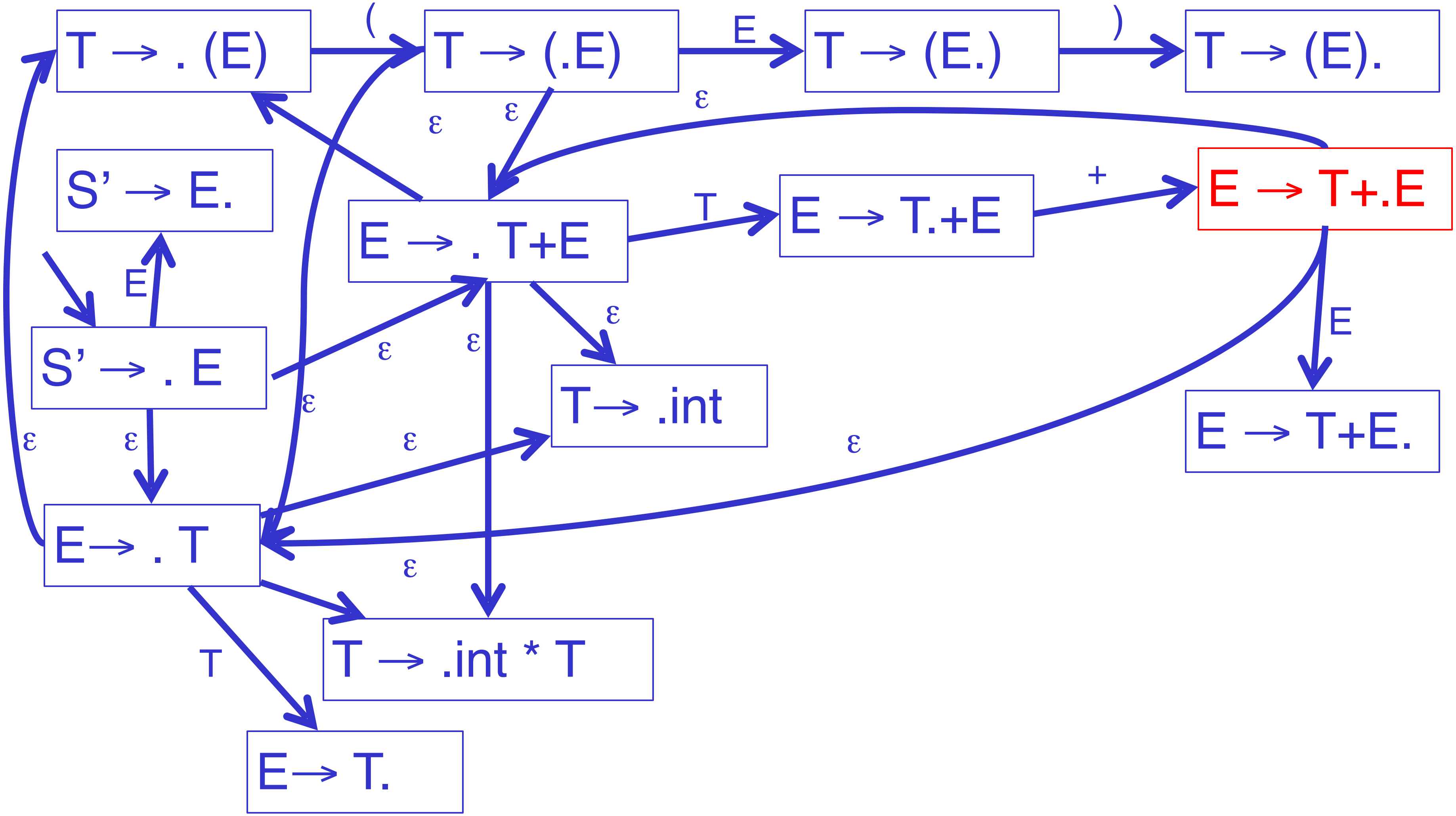
$T \rightarrow int * T \mid int \mid (E)$

NFA for Viable Prefixes



E → T + E | T
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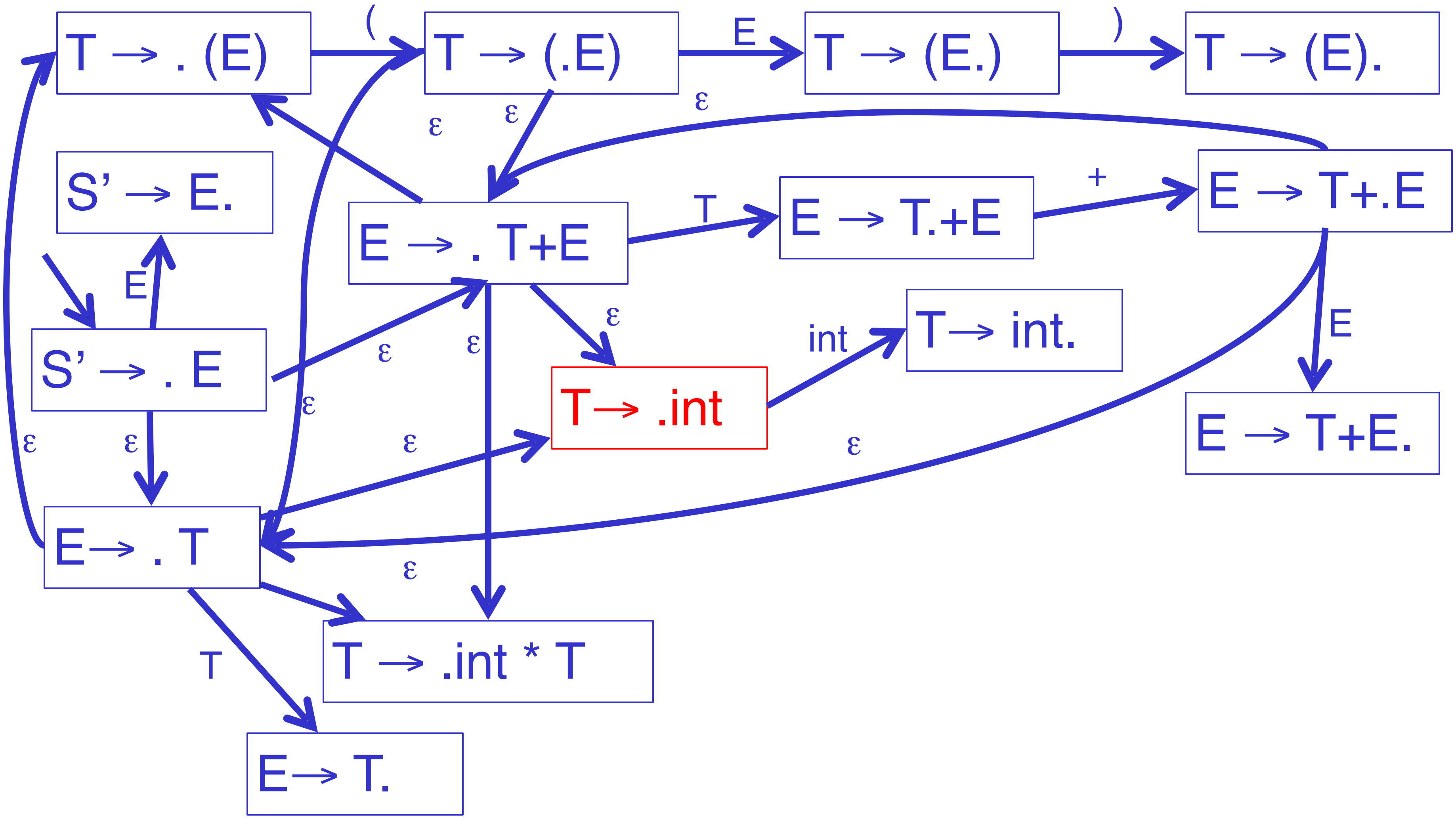
NFA for Viable Prefixes



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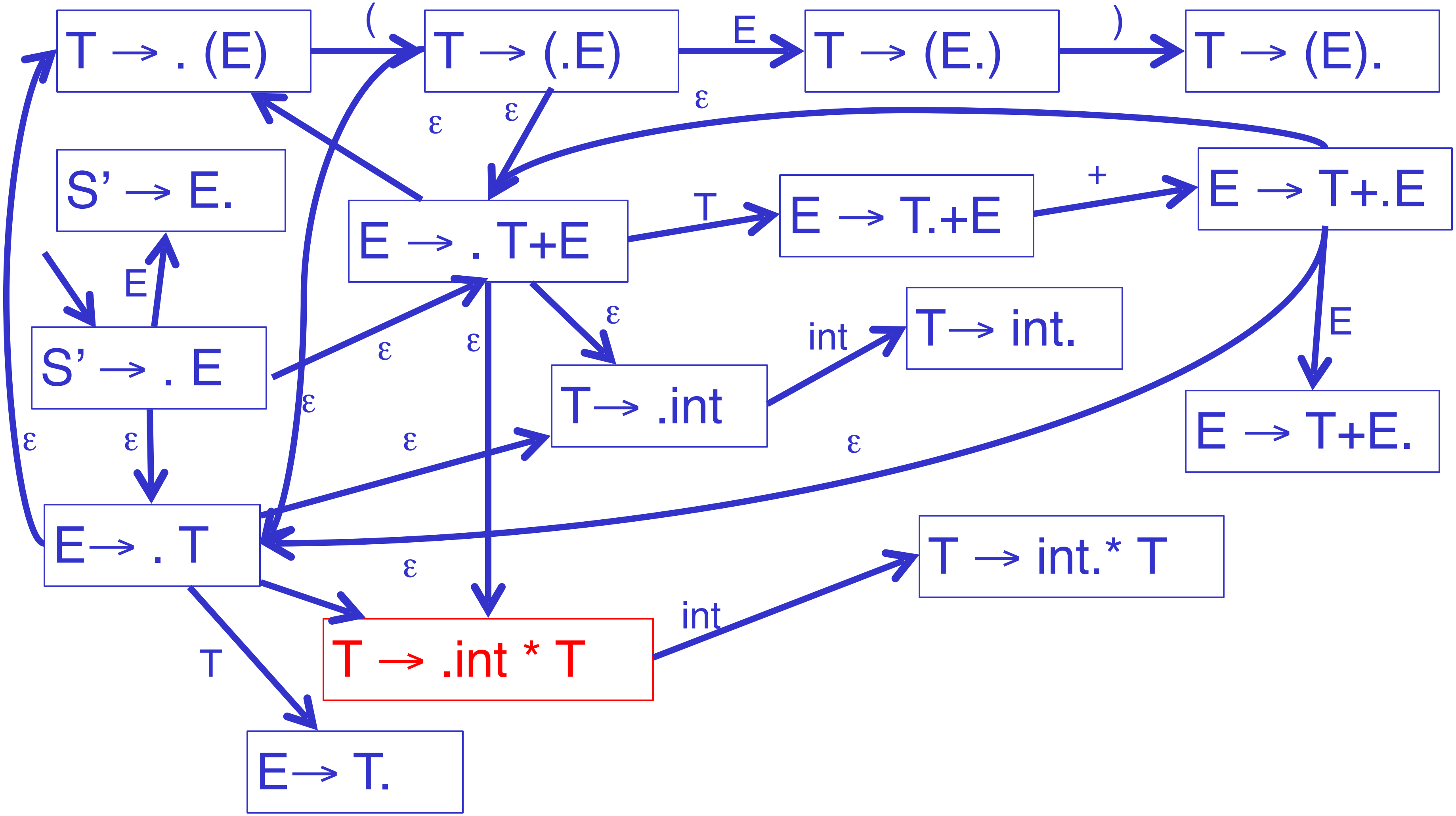
NFA for Viable Prefixes



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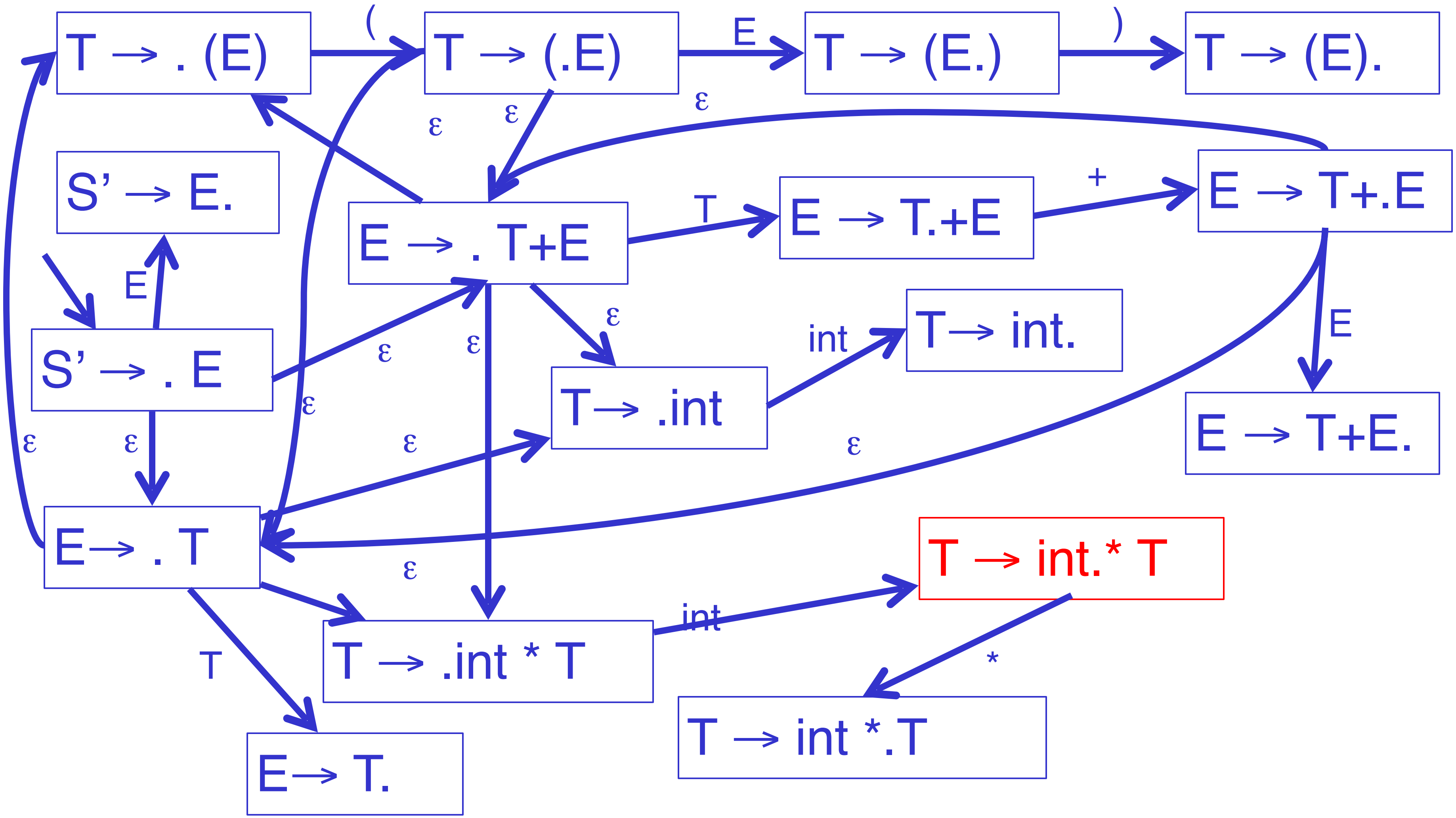
NFA for Viable Prefixes



E → T + E | T
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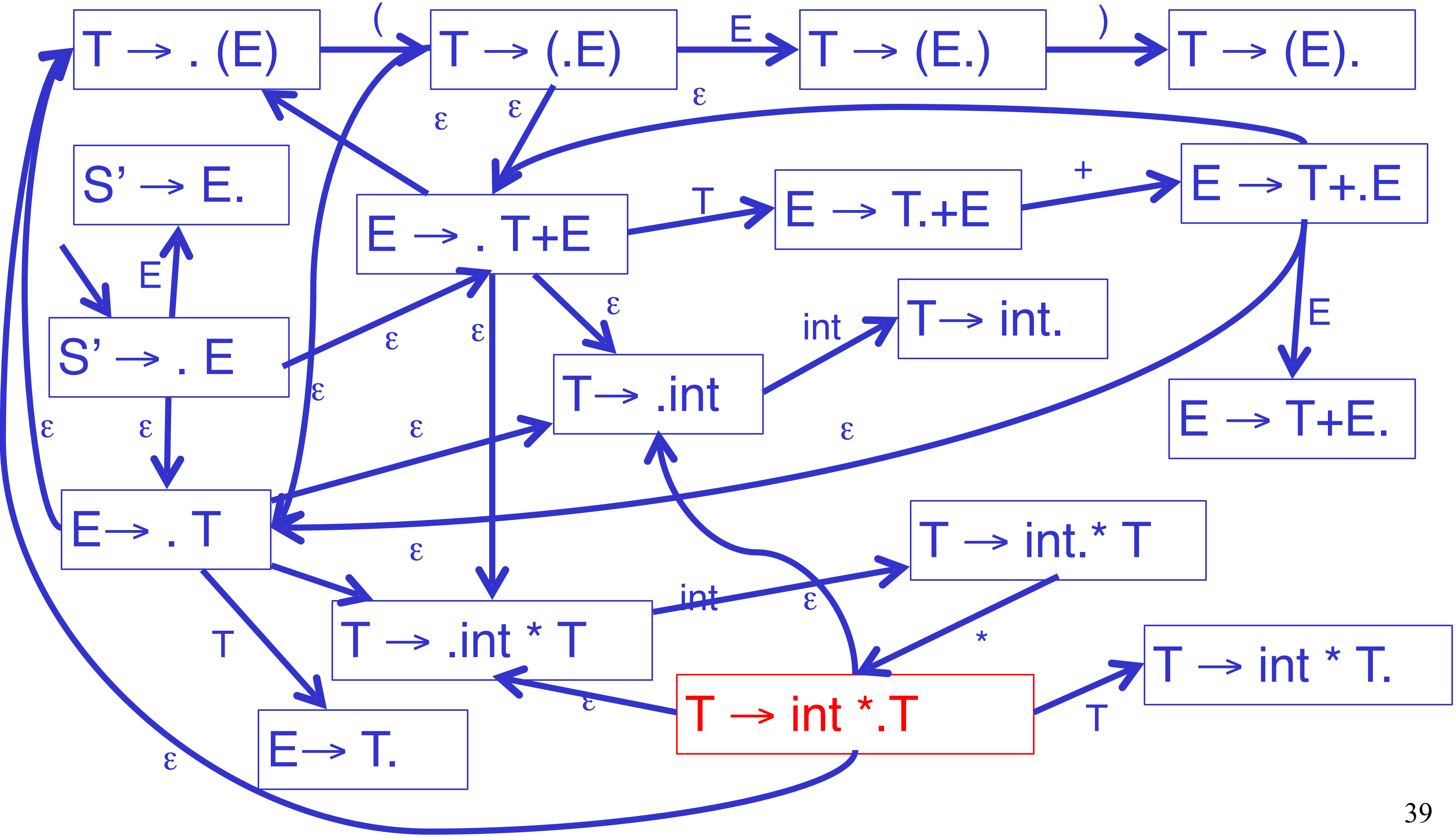
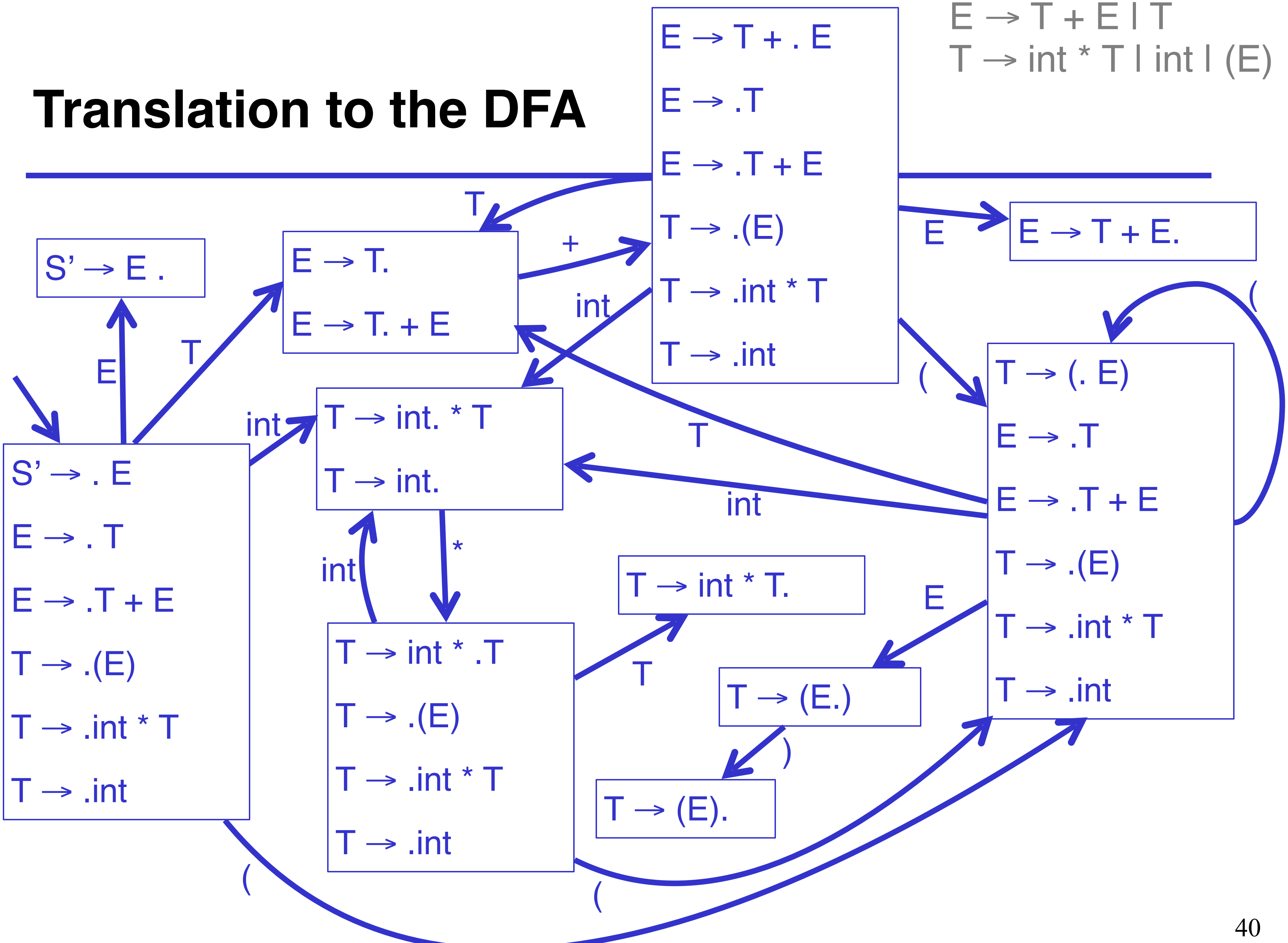
NFA for Viable Prefixes



Translation to the DFA

E → T + E | T
T → int * T | int | (E)



Lingo

The states of the DFA are

"canonical collections of items"

or

"canonical collections of LR(0) items"

The Dragon book gives another way of constructing LR(0) items

Valid Items

Item $X \rightarrow \beta.\gamma$ is valid for a viable prefix $\alpha\beta$ if

$$S' \rightarrow^* \alpha X \omega \rightarrow \alpha\beta\gamma\omega$$

by a right-most derivation

After parsing $\alpha\beta$, the valid items are the possible tops of the stack of items

Items Valid for a Prefix

An item I is valid for a viable prefix α if the DFA recognizing viable prefixes terminates on input α in a state s containing I

The items in s describe what the top of the item stack might be after reading input α

Valid Items Example

- An item is often valid for many prefixes

- Example: The item $T \rightarrow (.E)$ is valid for prefixes

  (

  ((

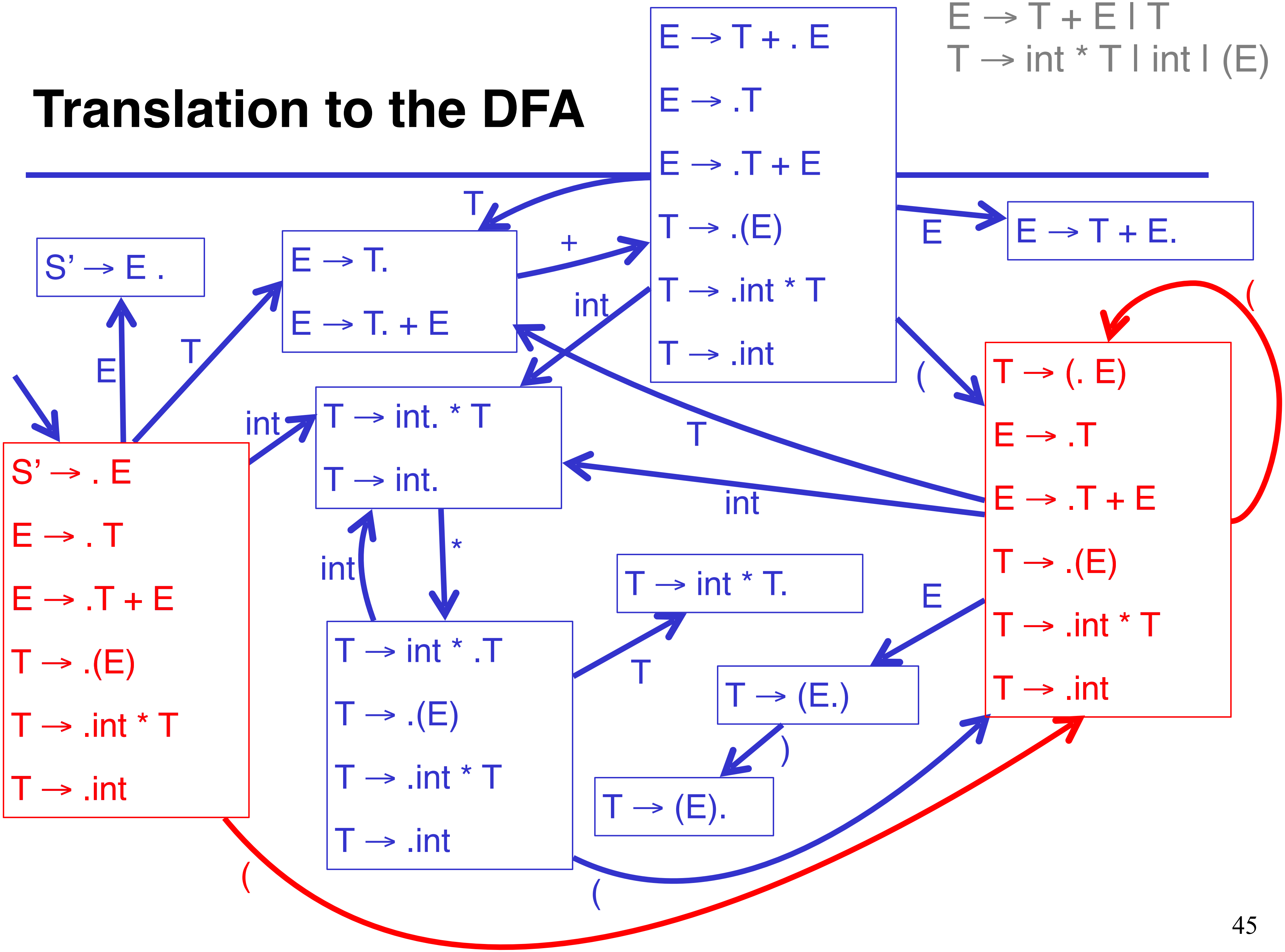
  (((

  ((((

  . . .

$E \rightarrow T + E \mid T$
$T \rightarrow int * T \mid int \mid (E)$

Translation to the DFA



LR(0) Parsing

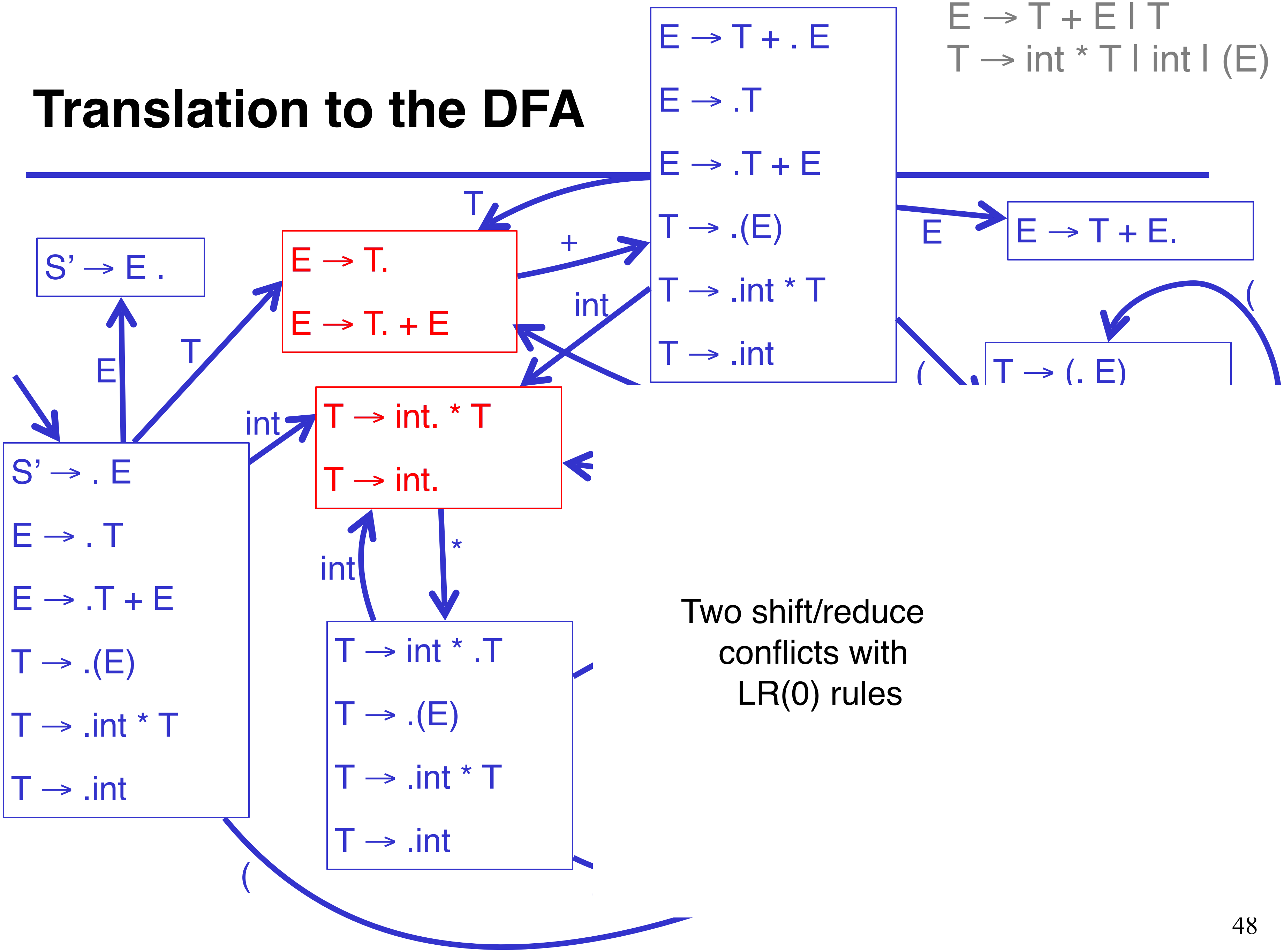
- Idea: Assume
  - stack contains α
  - next input is t
  - DFA on input α terminates in state s
- Reduce by $X \rightarrow \beta$ if
  - s contains item $X \rightarrow \beta.$
- Shift if
  - s contains item $X \rightarrow \beta.t\omega$
  - equivalent to saying s has a transition labeled t

LR(0) Conflicts

- LR(0) has a reduce/reduce conflict if:
  - Any state has two reduce items:
  - $X \rightarrow \beta.$ and $Y \rightarrow \omega.$
- LR(0) has a shift/reduce conflict if:
  - Any state has a reduce item and a shift item:
  - $X \rightarrow \beta.$ and $Y \rightarrow \omega.t\delta$

Translation to the DFA

E → T + E | T
T → int * T | int | (E)



Two shift/reduce conflicts with LR(0) rules

SLR

- LR = "Left-to-right scan"
- SLR = "Simple LR"

- SLR improves on LR(0) shift/reduce heuristics
  - Fewer states have conflicts

SLR Parsing

- Idea: Assume
  - stack contains α
  - next input is t
  - DFA on input α terminates in state s
- Reduce by $X \rightarrow \beta$ if
  - s contains item $X \rightarrow \beta.$
  - $t \in \text{Follow}(X)$ ←
- Shift if
  - s contains item $X \rightarrow \beta.t\omega$

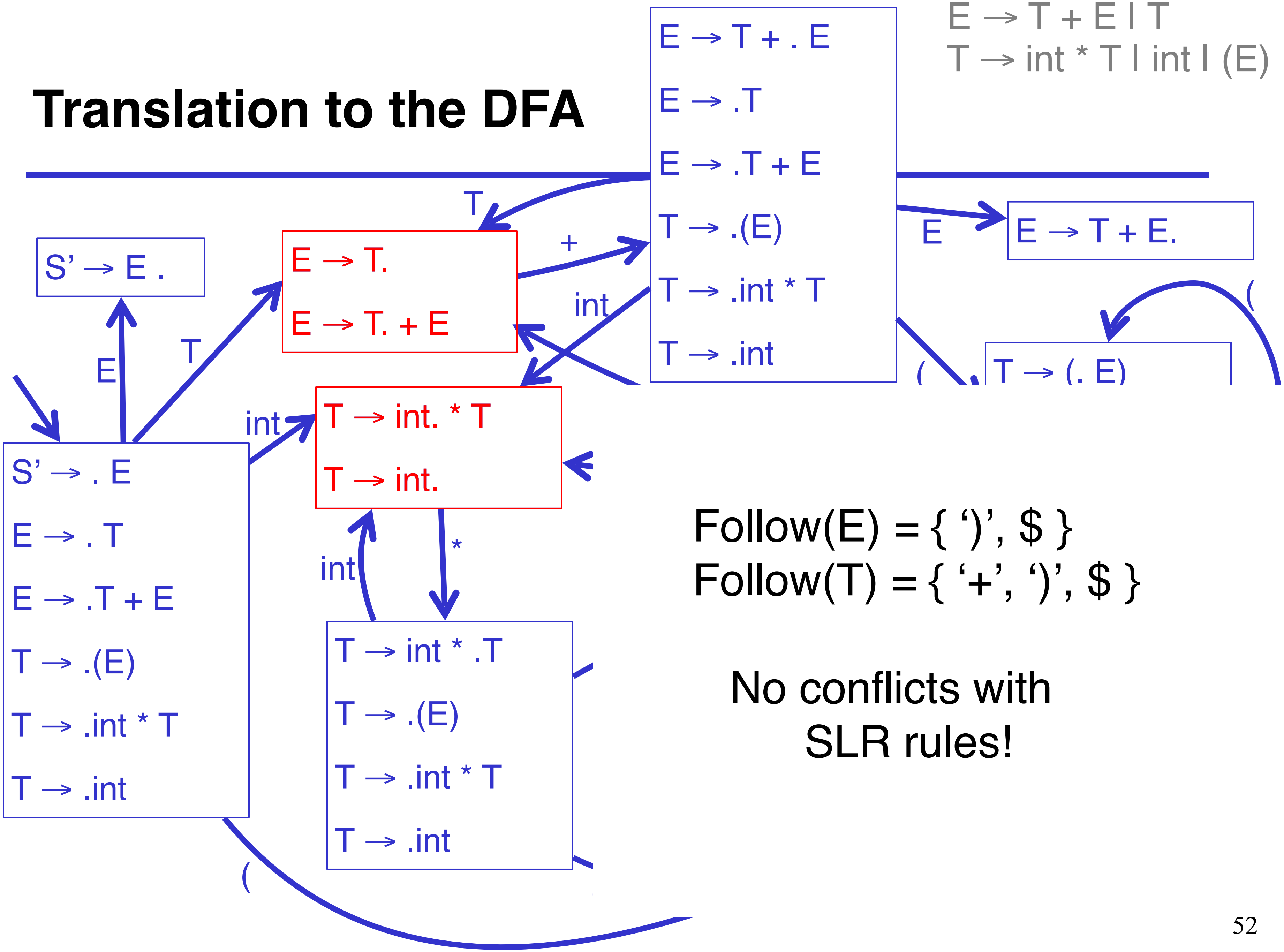
SLR Parsing (Cont.)

- If there are conflicts under these rules, the grammar is not SLR

- The rules amount to a heuristic for detecting handles
  - The SLR grammars are those where the heuristics detect exactly the handles

Translation to the DFA

E → T + E | T
T → int * T | int | (E)



Follow(E) = { ‘)’, $ }
Follow(T) = { ‘+’, ‘)’, $ }

No conflicts with SLR rules!

Precedence Declarations Digression

- Lots of grammars aren't SLR
  - including all ambiguous grammars
- We can parse more grammars by using precedence declarations
  - Instructions for resolving conflicts

Precedence Declarations (Cont.)

- Consider our favorite ambiguous grammar:
  - E → E + E | E * E | (E) | int
- The DFA for this grammar contains a state with the following items:
  - E → E * E .   E → E . + E
  - shift/reduce conflict!
- Declaring "* has higher precedence than +" resolves this conflict in favor of reducing

Precedence Declarations (Cont.)

- The term “precedence declaration” is misleading
- These declarations do not define precedence; they define conflict resolutions
  - Not quite the same thing!

Naïve SLR Parsing Algorithm

1. Let M be DFA for viable prefixes of G
2. Let $|x_1 \ldots x_n\$$ be initial configuration
3. Repeat until configuration is $S|\$$
   - Let $\alpha|\omega$ be current configuration
   - Run M on current stack α
   - If M rejects α, report parsing error
     - Stack α is not a viable prefix
   - If M accepts α with items I, let t be next input
     - Reduce if $X \rightarrow \beta. \in I$ and $t \in Follow(X)$
     - Otherwise, shift if $X \rightarrow \beta.\ t\ \gamma \in I$
     - Report parsing error if neither applies

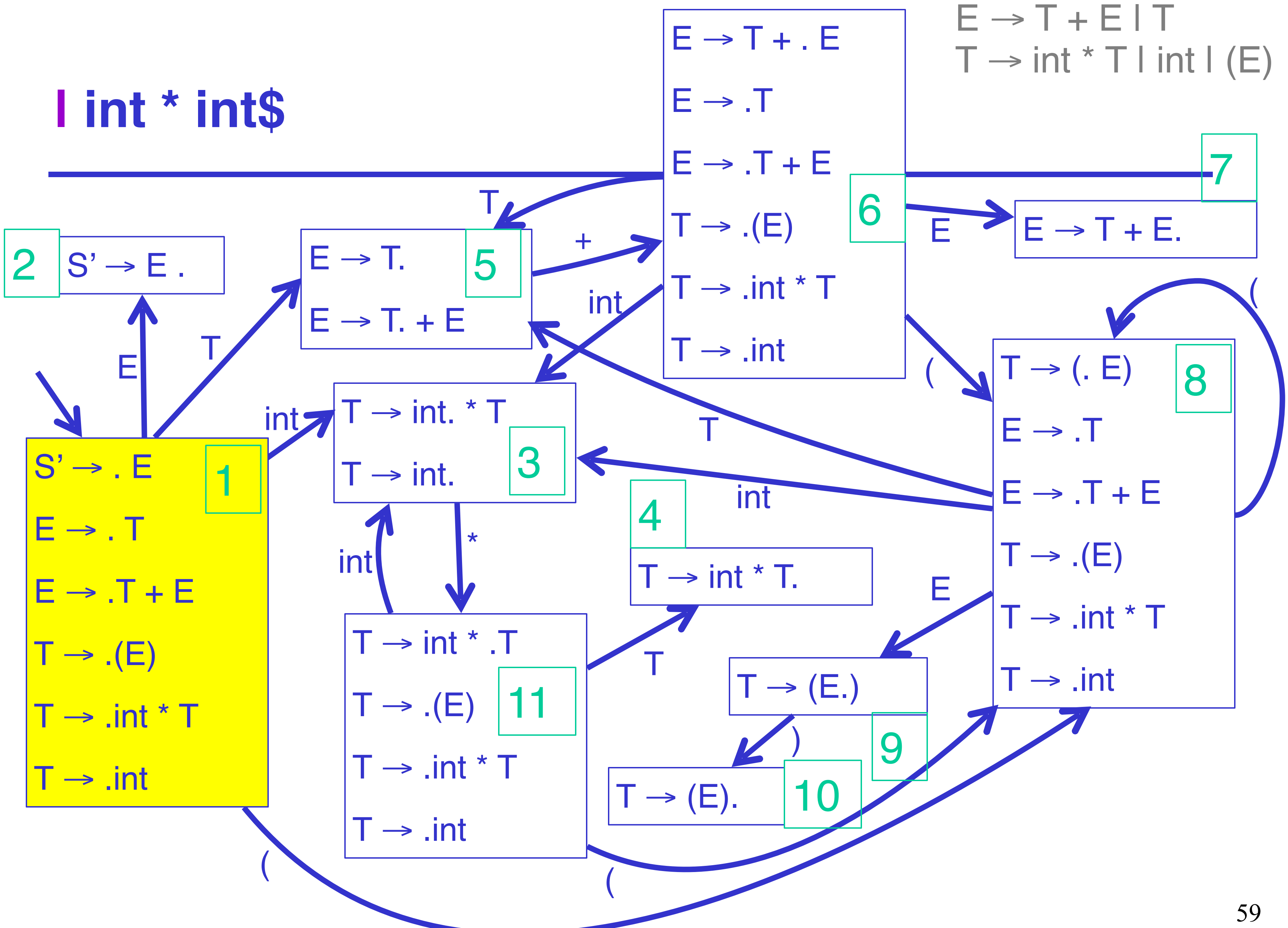
Notes

- If there is a conflict in the last step, grammar is not SLR(k)
- k is the amount of lookahead
  - In practice k = 1
- Will skip using extra start state S' in following example to save space on slides

$E \rightarrow T + E \mid T$
$T \rightarrow int * T \mid int \mid (E)$

SLR Example

Configuration	DFA Halt State	Action
| int * int$	1	shift



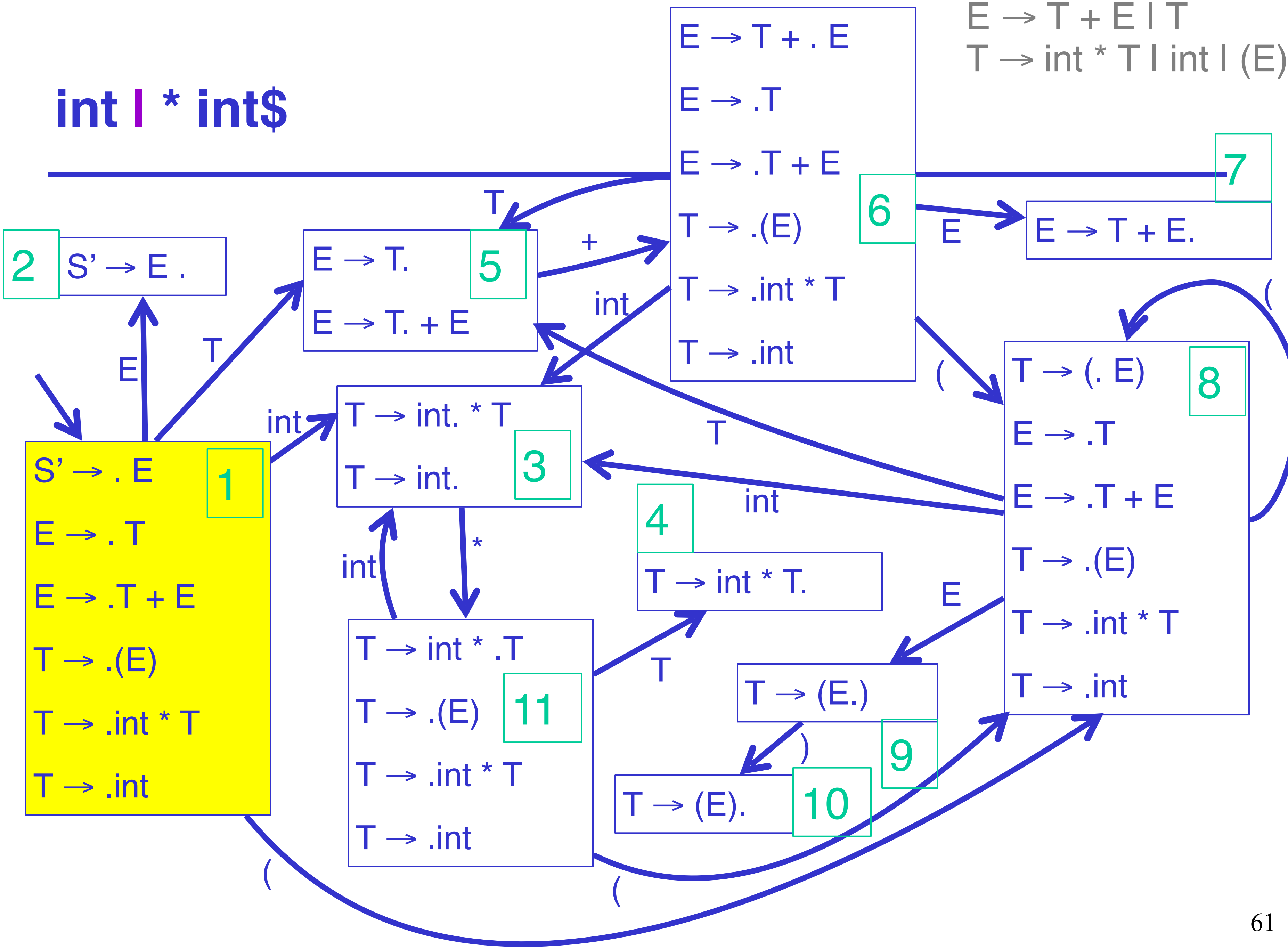
E → T + E | T
T → int * T | int | (E)
| int * int$
1
S' → . E
E → . T
E → .T + E
T → .(E)
T → .int * T
T → .int
2
S' → E .
3
T → int. * T
T → int.
4
T → int * T.
5
E → T.
E → T. + E
6
E → T + . E
E → .T
E → .T + E
T → .(E)
T → .int * T
T → .int
7
E → T + E.
8
T → (. E)
E → .T
E → .T + E
T → .(E)
T → .int * T
T → .int
9
T → (E.)
10
T → (E).
11
T → int * .T
T → .(E)
T → .int * T
T → .int
E
T
int
+
*
(
)

SLR Example

Configuration	DFA Halt State	Action
| int * int$	1	shift
int | * int$	3 * not in Follow(T)	shift

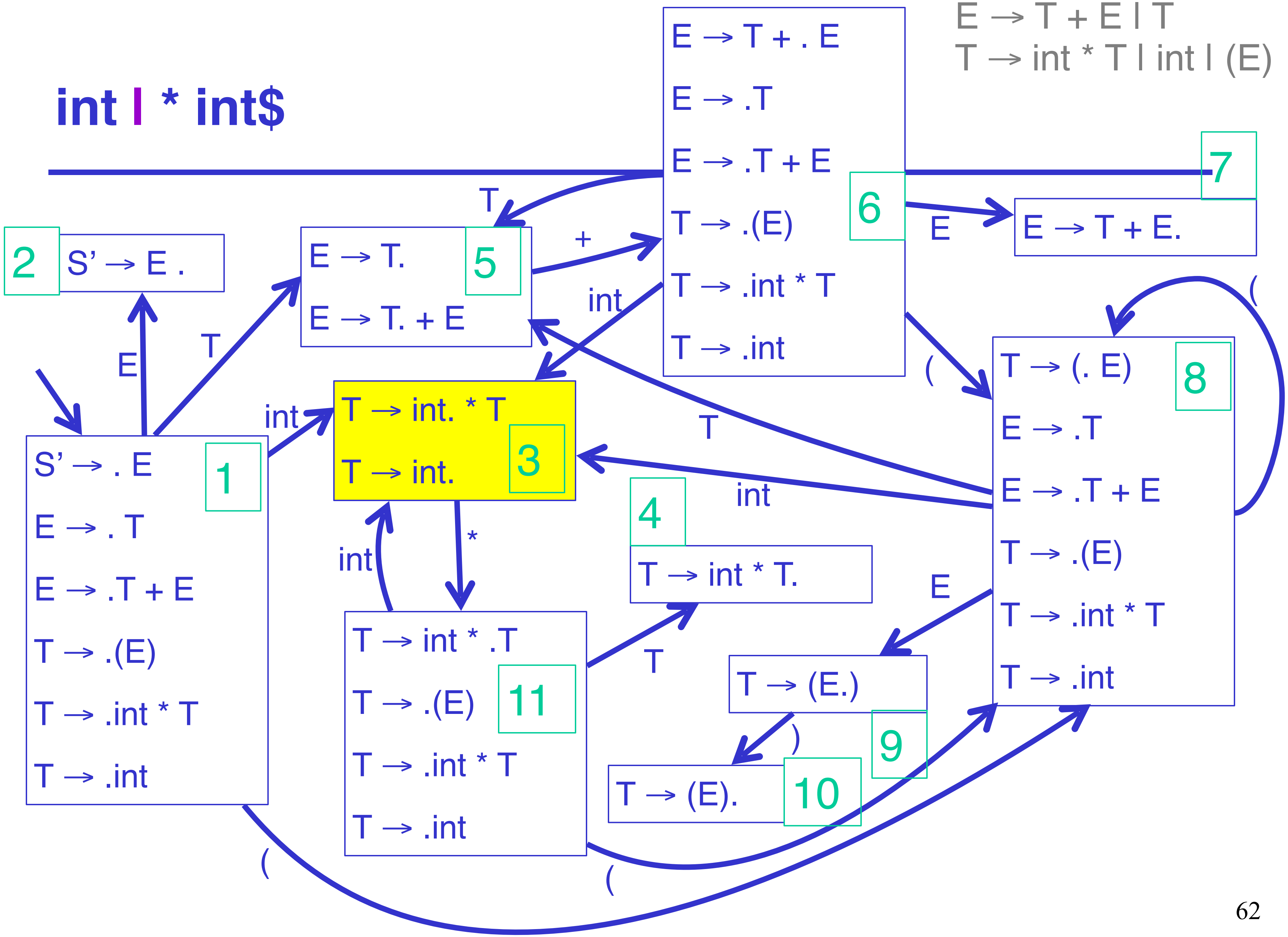
E → T + E | T
T → int * T | int | (E)

int | * int$



E → T + E | T
T → int * T | int | (E)

int | * int$



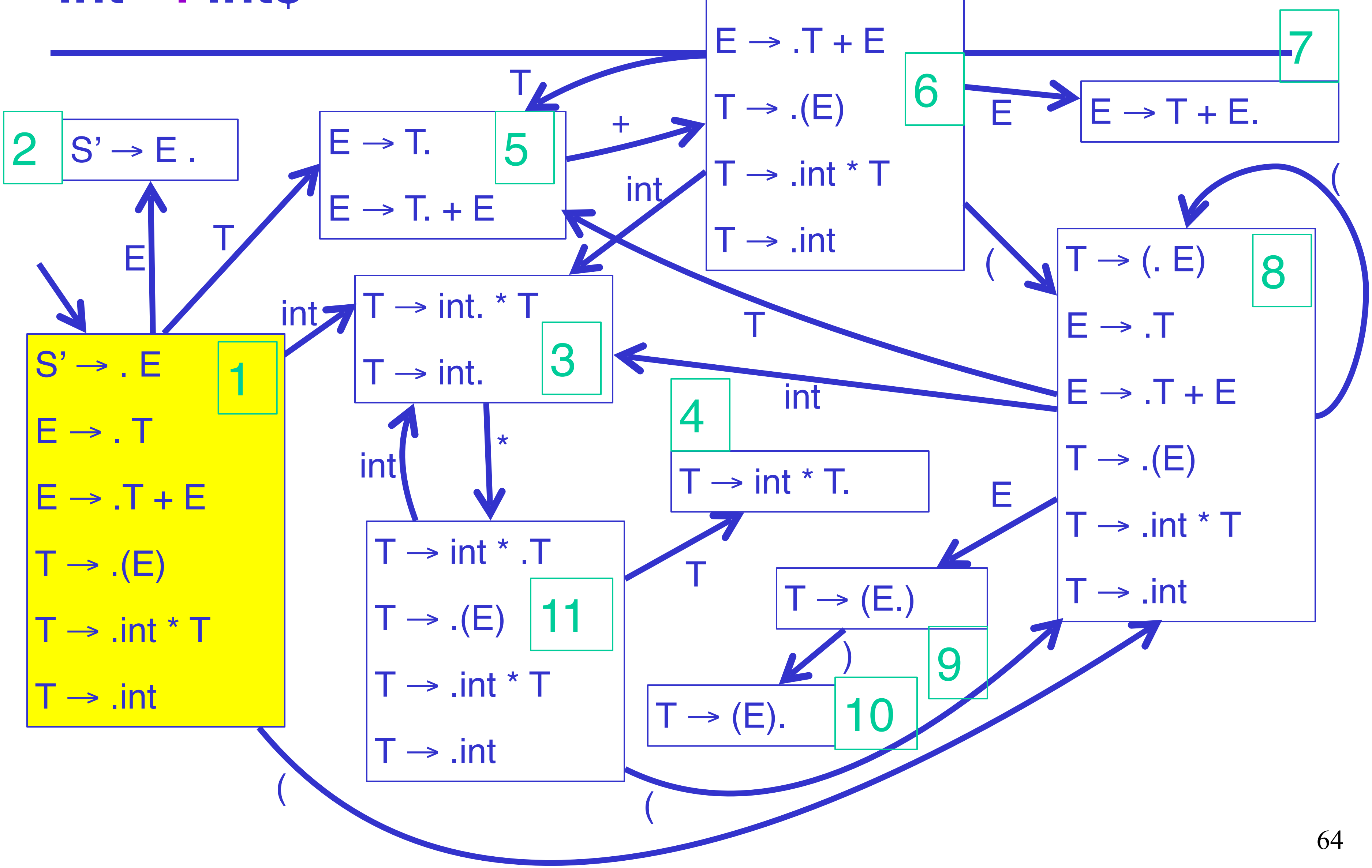
$E \rightarrow T + E \mid T$
$T \rightarrow int * T \mid int \mid (E)$

SLR Example

Configuration	DFA Halt State	Action
| int * int$	1	shift
int | * int$	3 * not in Follow(T)	shift
int * | int$	11	shift

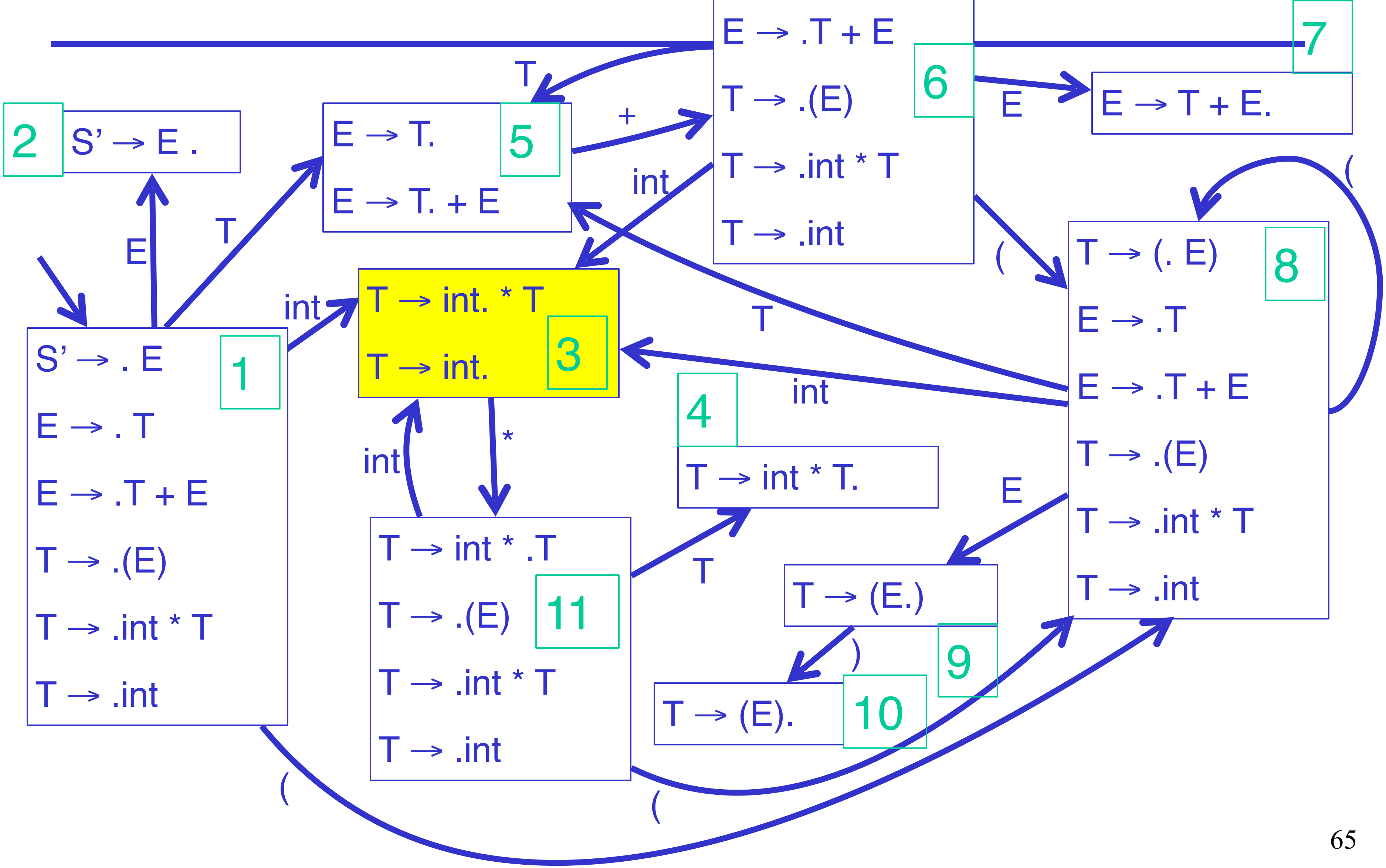
E → T + E | T
T → int * T | int | (E)

int * | int$



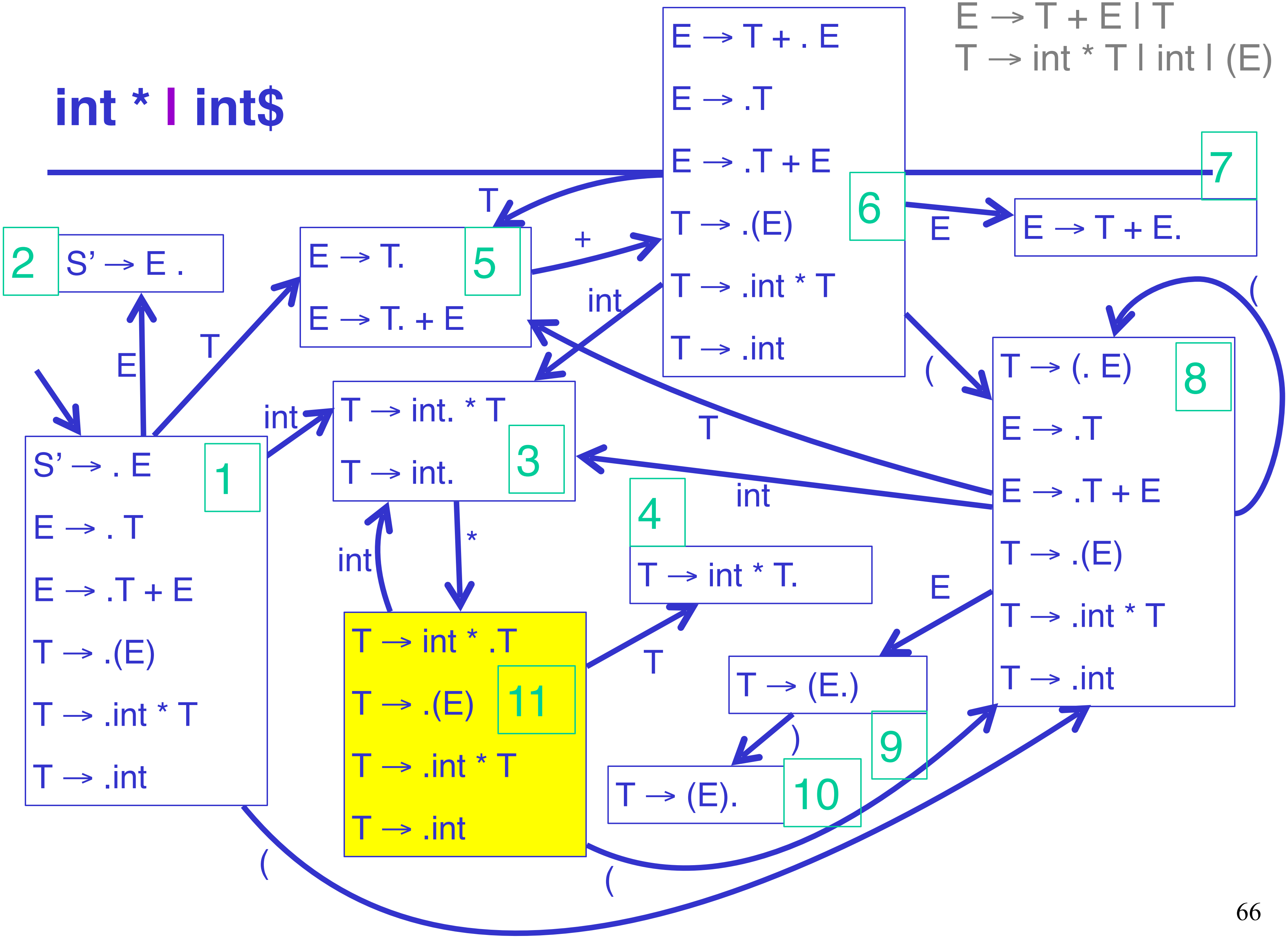
int * | int$

E → T + E | T
T → int * T | int | (E)



E → T + E | T
T → int * T | int | (E)

int * | int$



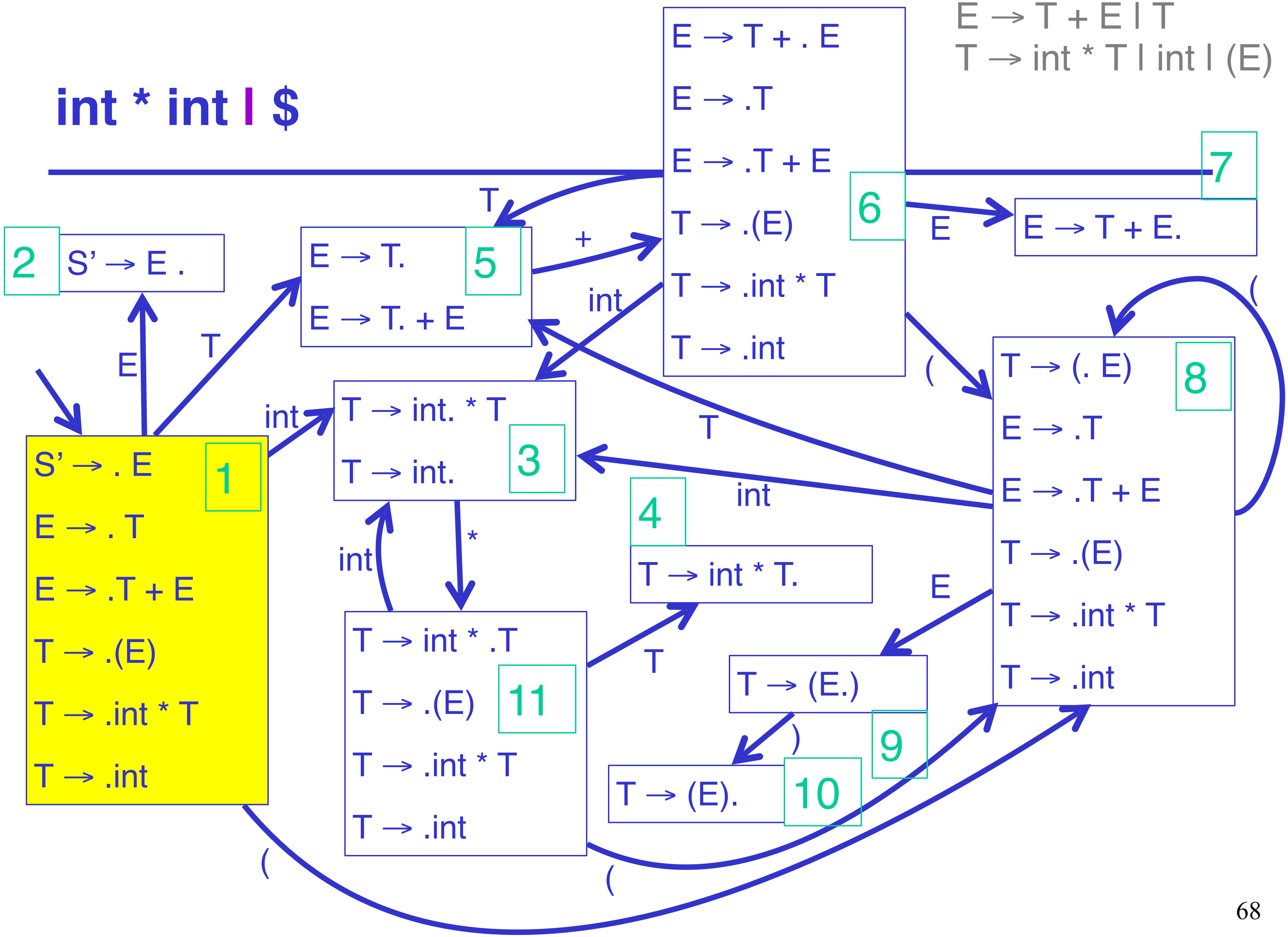
$E \rightarrow T + E \mid T$
$T \rightarrow int * T \mid int \mid (E)$

SLR Example

Configuration	DFA Halt State		Action
I int * int$	1		shift
int I * int$	3	* not in Follow(T)	shift
int * I int$	11		shift
int * int I$	3	$ ∈ Follow(T)	reduce T→int

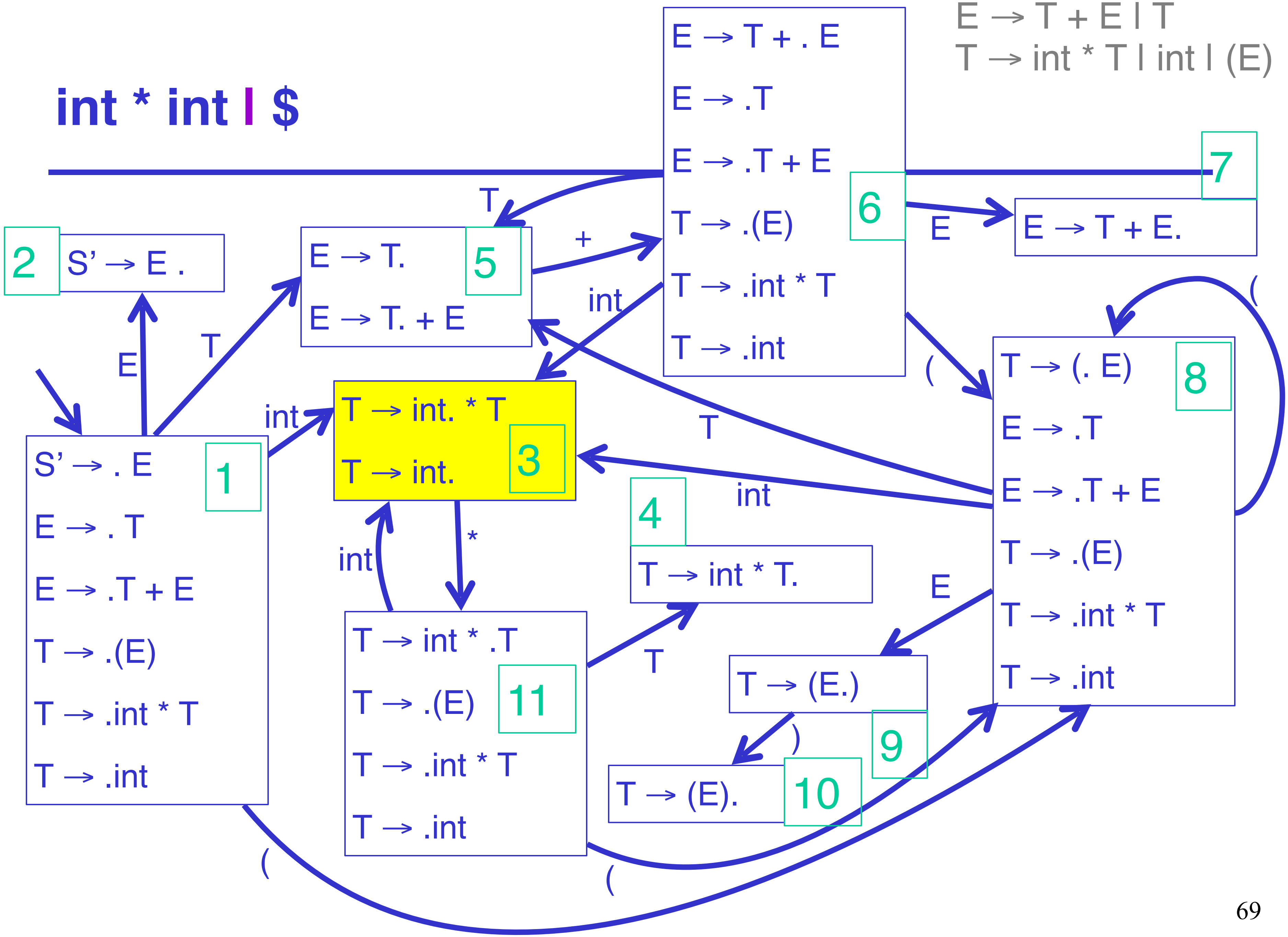
E → T + E | T
T → int * T | int | (E)

int * int | $



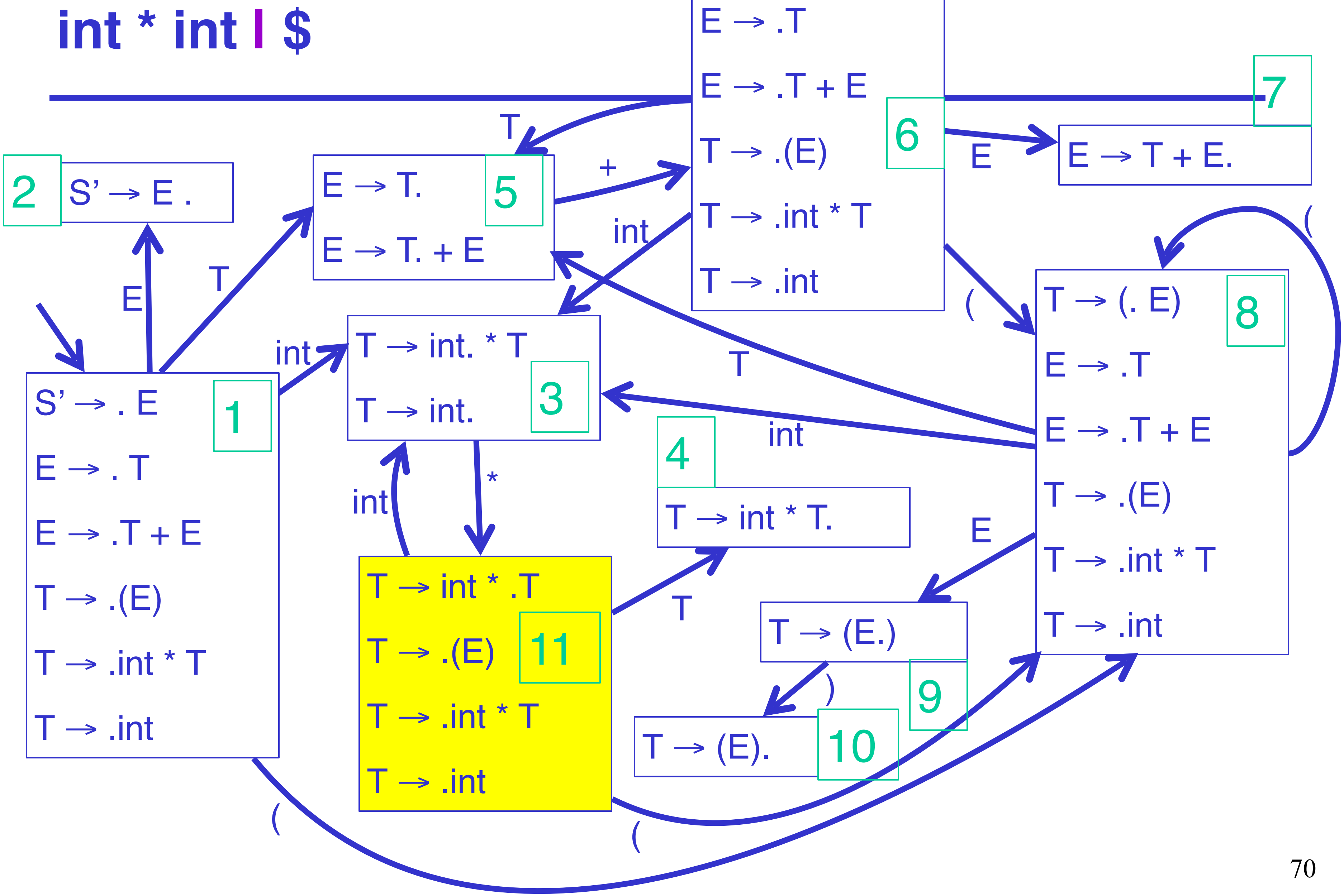
E → T + E | T
T → int * T | int | (E)

int * int | $



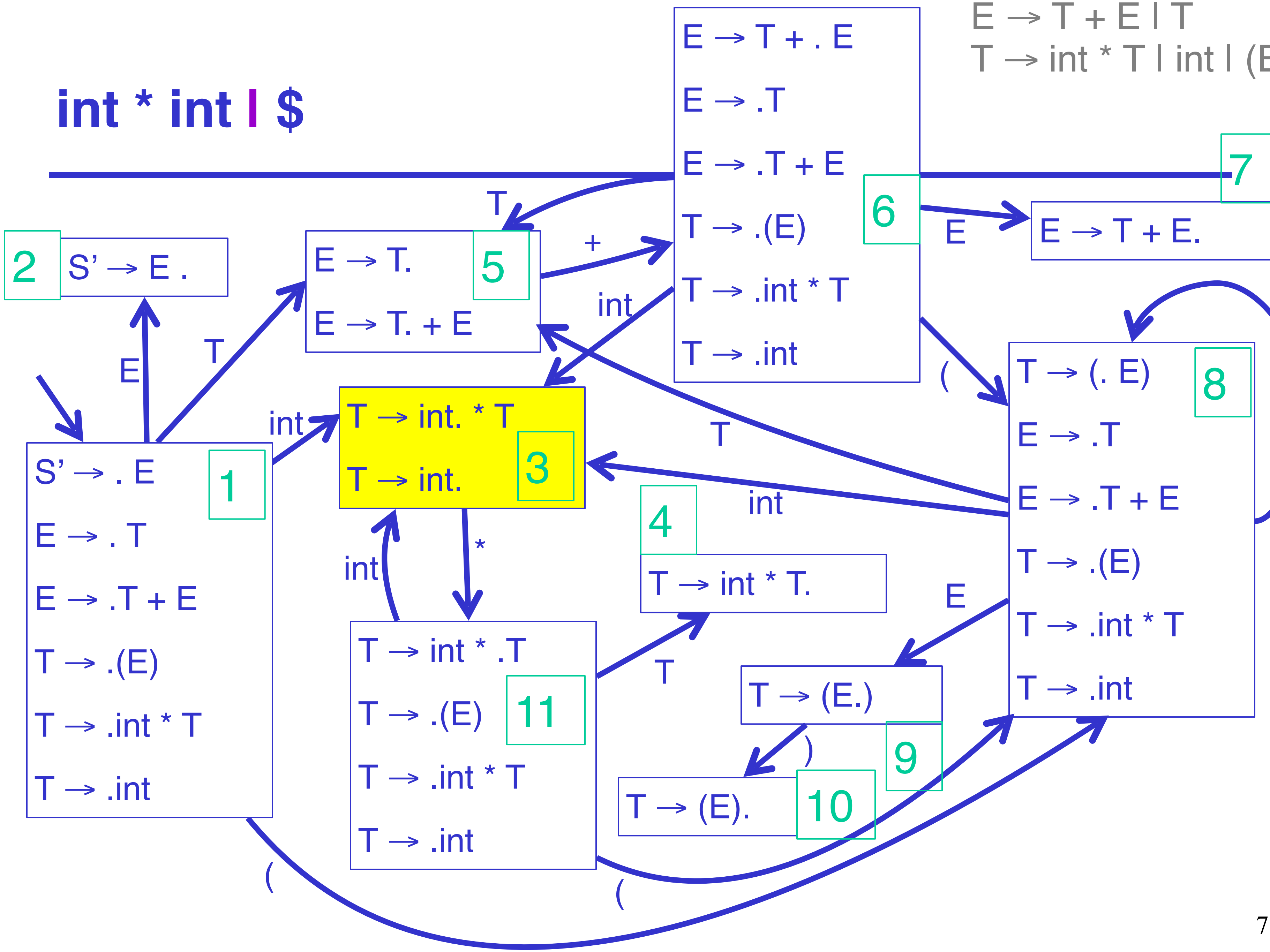
E → T + E | T
T → int * T | int | (E)

int * int | $



int * int | $

E → T + E | T
T → int * T | int | (E)



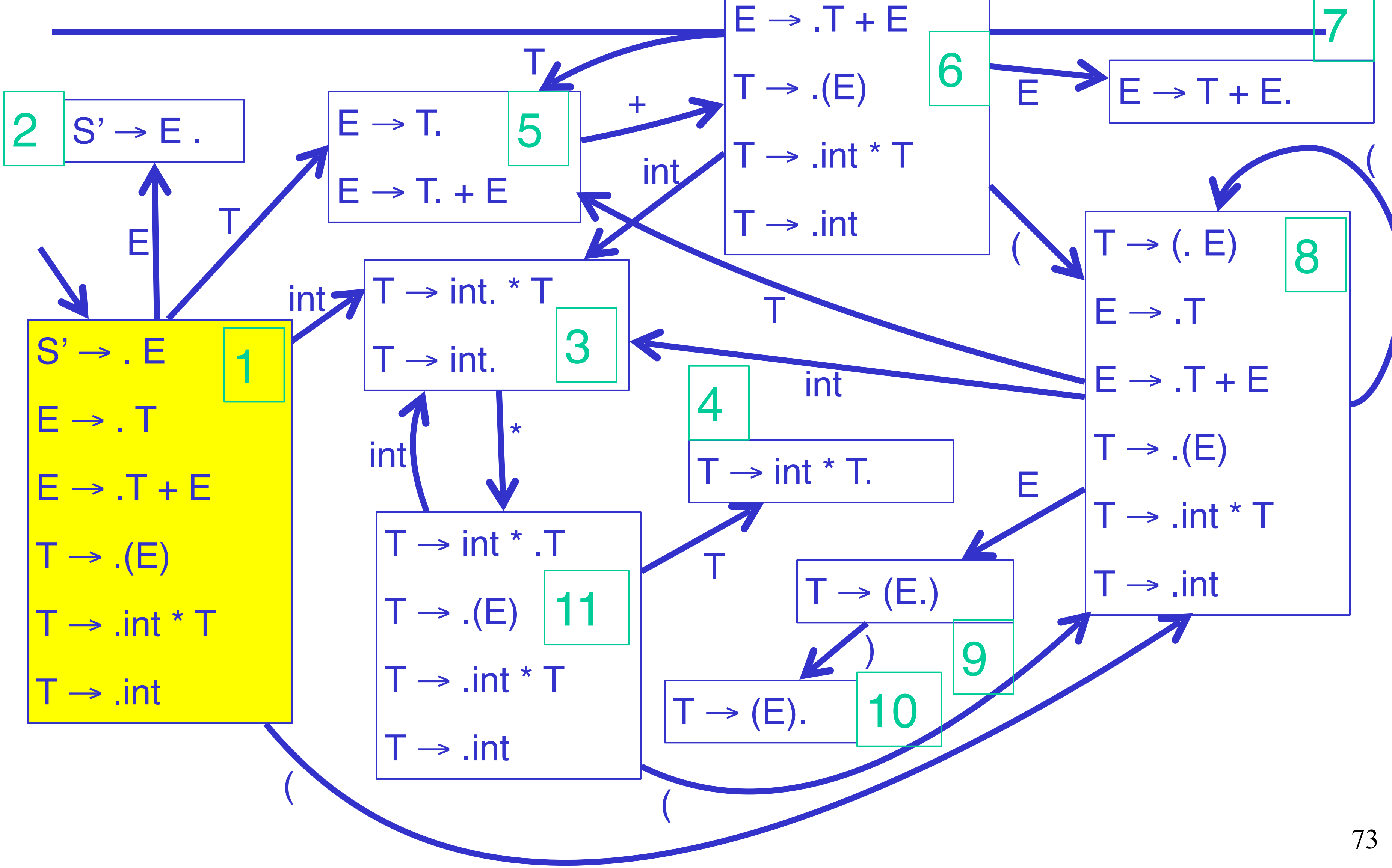
$E \rightarrow T + E \mid T$
$T \rightarrow int * T \mid int \mid (E)$

SLR Example

Configuration	DFA Halt State		Action
I int * int$	1		shift
int I * int$	3	* not in Follow(T)	shift
int * I int$	11		shift
int * int I$	3	$ ∈ Follow(T)	reduce T→int
int * T I$	4	$ ∈ Follow(T)	reduce T→int*T

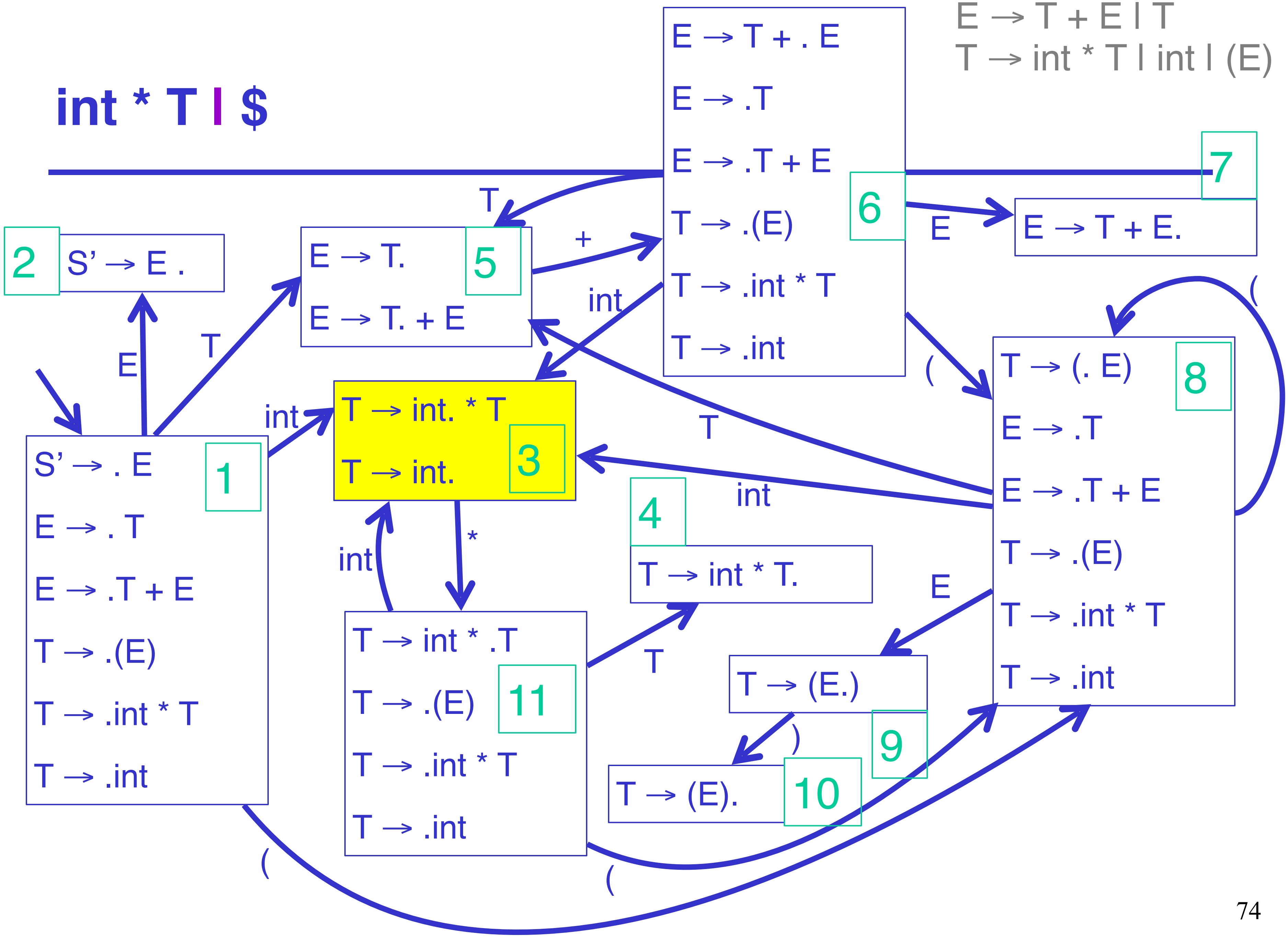
E → T + E | T
T → int * T | int | (E)

int * T | $



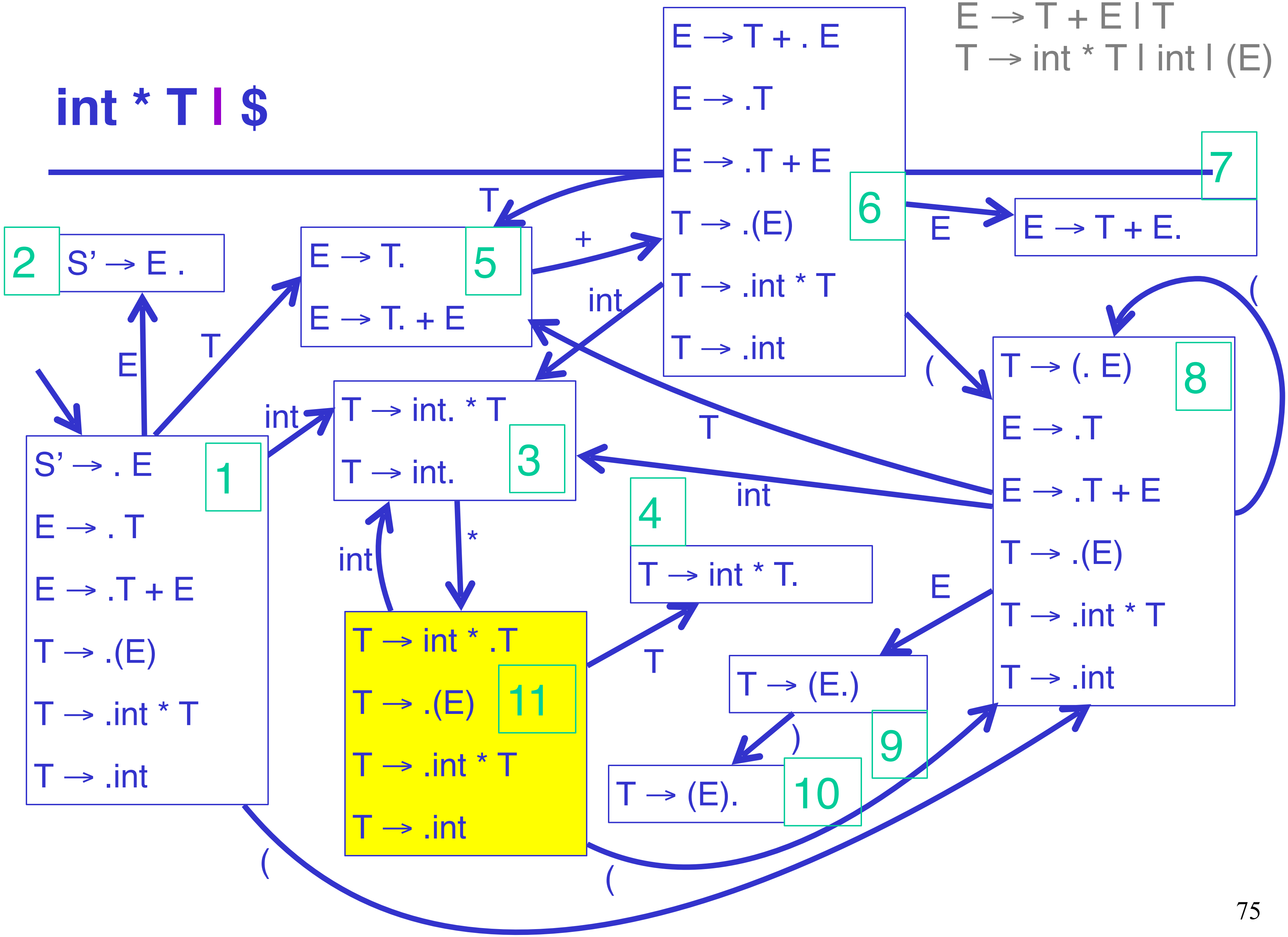
E → T + E | T
T → int * T | int | (E)

int * T | $



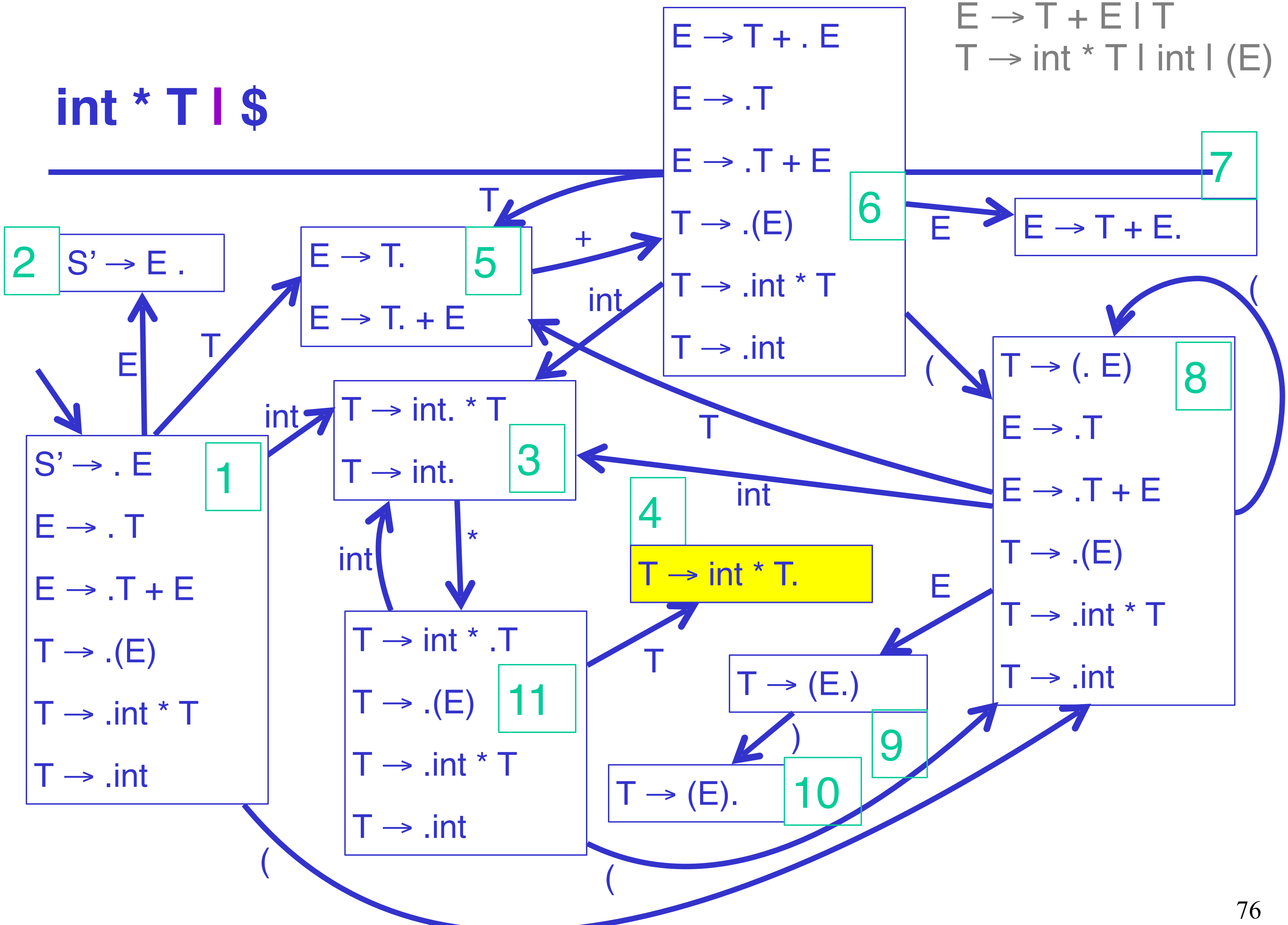
E → T + E | T
T → int * T | int | (E)

int * T | $



E → T + E | T
T → int * T | int | (E)

int * T | $



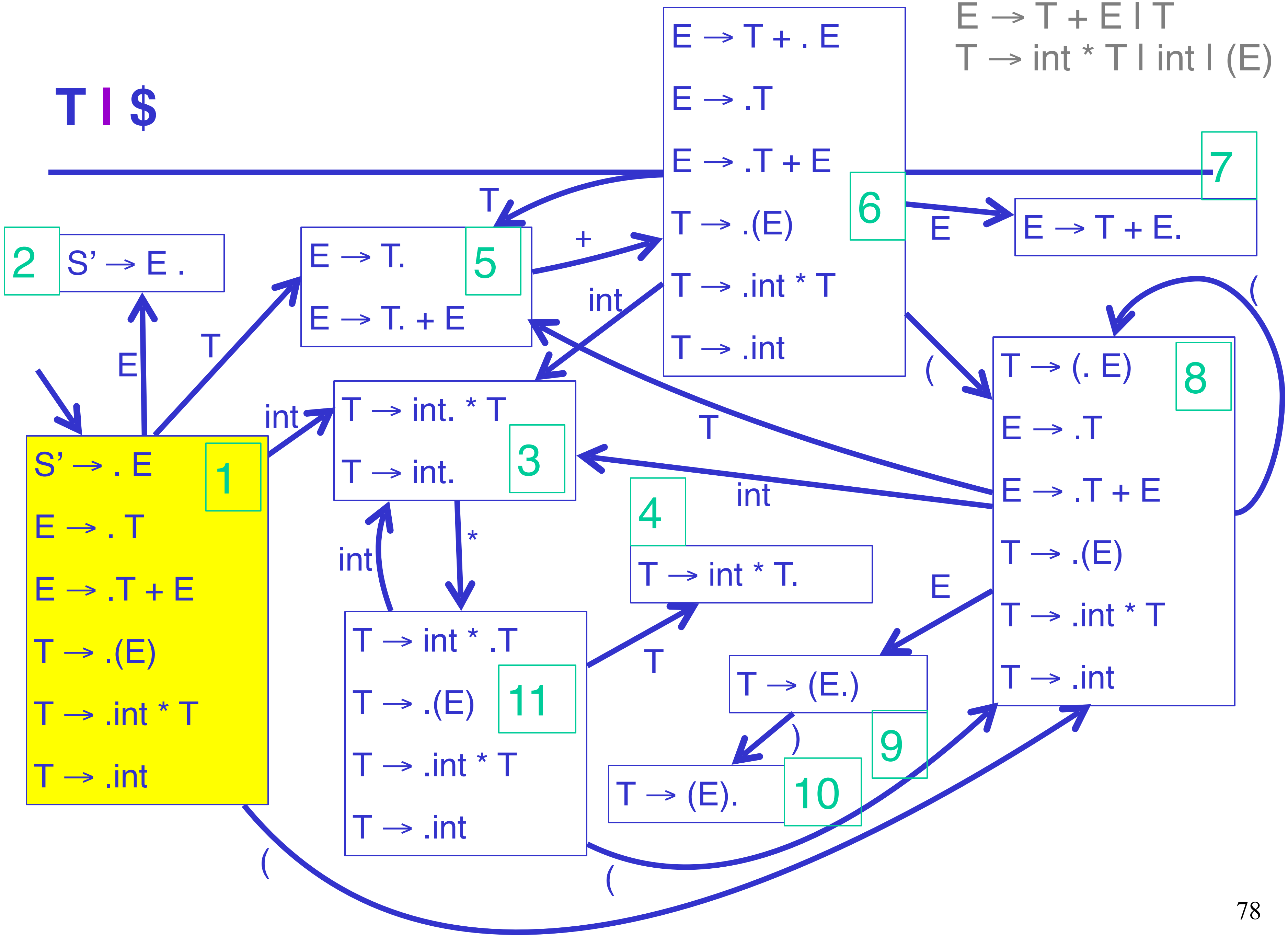
E → T + E | T
T → int * T | int | (E)

SLR Example

Configuration	DFA Halt State		Action
| int * int$	1		shift
int | * int$	3	* not in Follow(T)	shift
int * | int$	11		shift
int * int |$	3	$ ∈ Follow(T)	reduce T→int
int * T |$	4	$ ∈ Follow(T)	reduce T→int*T
T |$	5	$ ∈ Follow(T)	reduce E→T

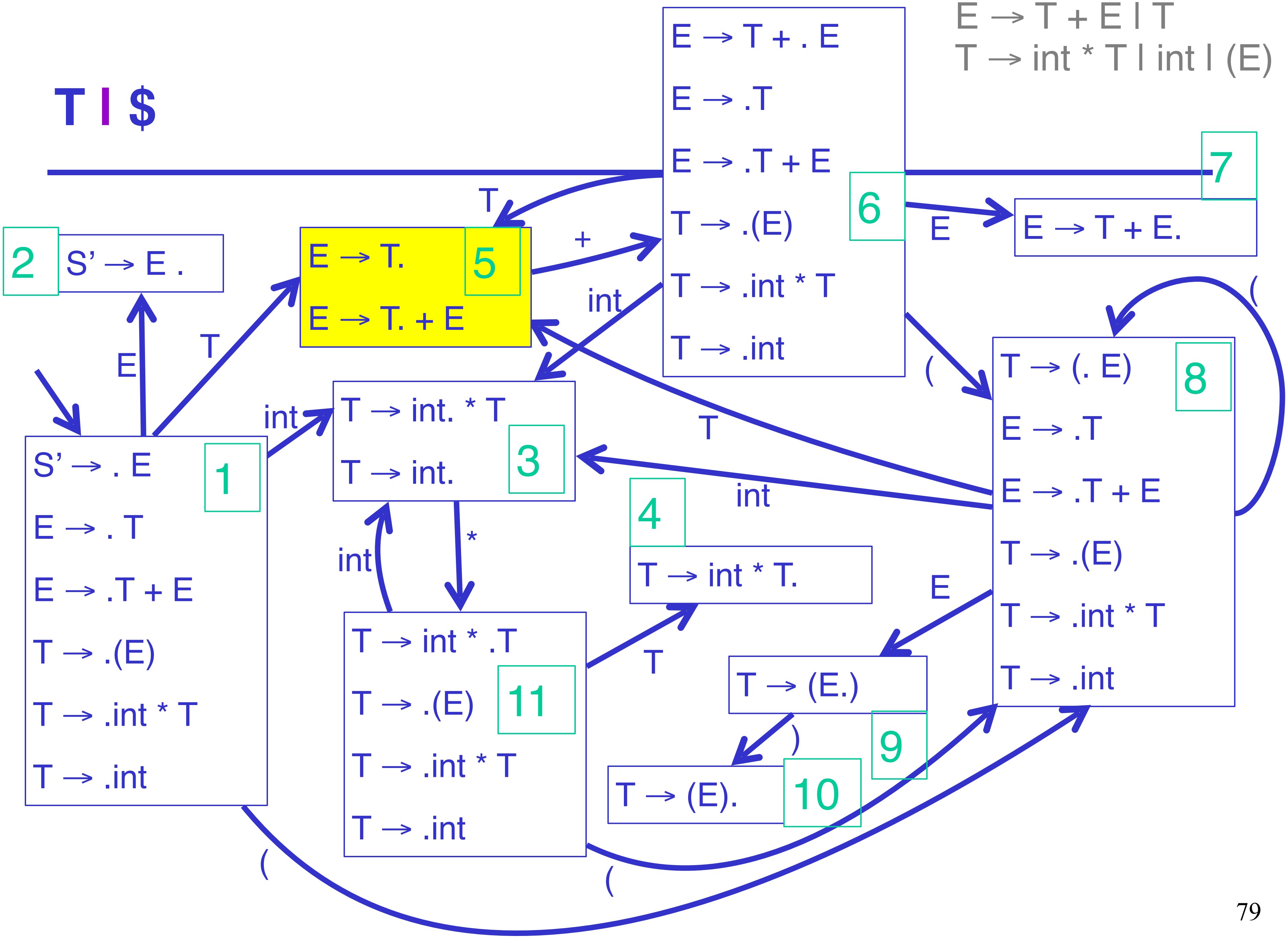
E → T + E | T
T → int * T | int | (E)

T | $



E → T + E | T
T → int * T | int | (E)

T | $



$E \to T + E \mid T$
$T \to \text{int} * T \mid \text{int} \mid (E)$

SLR Example

Configuration	DFA Halt State		Action
I int * int$	1		shift
int I * int$	3	* not in Follow(T)	shift
int * I int$	11		shift
int * int I$	3	$ ∈ Follow(T)	reduce T→int
int * T I$	4	$ ∈ Follow(T)	reduce T→int*T
T I$	5	$ ∈ Follow(T)	reduce E→T
E I$			accept

An Improvement

- Rerunning the automaton at each step is wasteful
  - Most of the work is repeated
- Remember the state of the automaton on each prefix of the stack
- Change stack to contain pairs

$$\langle \text{ symbol, DFA state } \rangle$$

An Improvement (Cont.)

- For a stack

  $\langle$ $symbol_1$, $state_1$ $\rangle$ . . . $\langle$ $symbol_n$, $state_n$ $\rangle$

  $state_n$ is the final state of the DFA on $symbol_1 \ldots symbol_n$

- Detail: The bottom of the stack is $\langle$dummy,start$\rangle$ where
  - dummy is a dummy symbol
  - start is the start state of the DFA

Goto (DFA) Table

- Define goto[i,A] = j if $state_i \rightarrow^{A} state_j$

- goto is just the transition function of the DFA
  - One of two parsing tables

Refined Parser Moves

- Shift x
  - Push $\langle a, x \rangle$ on the stack
  - a is current input
  - x is a DFA state
- Reduce $X \rightarrow \alpha$
  - As before
- Accept
- Error

Action Table

For each state s_i and terminal t

- If s_i has item $X \rightarrow \alpha.t\beta$ and goto[i,t] = k
  then action[i,t] = shift k

- If s_i has item $X \rightarrow \alpha.$ and $t \in$ Follow(X) and $X \neq S'$ then
  action[i,t] = reduce $X \rightarrow \alpha$

- If s_i has item $S' \rightarrow S.$ then action[i,$] = accept

- Otherwise, action[i,t] = error

SLR Parsing Algorithm

Let input = w$ be initial input

Let j = 0

Let DFA state 1 be the one with item S' → .S

Let stack = ⟨ dummy, 1 ⟩ // ⟨ symbol, state ⟩

repeat

case action[top_state(stack), input[j]] of

shift k: push ⟨ input[j++], k ⟩

reduce X → α:

pop $|\alpha|$ pairs,

push ⟨X, goto[top_state(stack), X]⟩

accept: halt normally

error: halt and report error

Notes on SLR Parsing Algorithm

- Note that the algorithm uses only the DFA states and the input
  - The stack symbols are never used!
- However, we still need the symbols for semantic actions

More Notes

- Some common constructs are not SLR(1)

- LR(1) is more powerful
  - Build lookahead into the items
  - An LR(1) item is a pair: (LR(0) item, x lookahead)
  - [T→ . int * T, $] means
    - After seeing T→ int * T reduce if lookahead is $
  - More accurate than just using follow sets
  - See Dragon Book
  - Take a look at the LR(1) automaton for your parser!